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A robust method to recognize critical configuration for camera calibration

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Abstract

When space points and camera optical center lie on a twisted cubic, no matter how many pairs there are used from the space points to their image points, camera parameters cannot be determined uniquely. This configuration is critical for camera calibration. We set up invariant relationship between six space points and their image points for the critical configuration. Then based on the relationship, an algorithm to recognize the critical configuration of at least six pairs of space and image points is proposed by using a constructed criterion function, where no any explicit computation on camera projective matrix or optical center is needed. Experiments show the efficiency of the proposed method.

Keywords: Camera calibration; Invariant; Critical configuration

1. Introduction

Projective geometric invariant plays an important role in computer vision. Since 1994, there have been many studies on the invariant relationship between six space points and their image points [2,4,5,7–11]. The invariant relationship can be applied to 3D reconstruction, object recognition, robot vision and so on as shown in the literature. Motivated by these works and the importance of invariants, we set up invariant relationships for two special configurations when six space points and camera optical center lie on a quadric cone or a twisted cubic. The configuration is called quadric cone or twisted cubic configuration. It is interesting that the number of equations describing the invariant relationship for the twisted cubic configuration is two and not one anymore. The number of equations describing for the general configuration in the previous literature is one. In addition, the invariant relationship in the previous work, and the ones here for the quadric cone and twisted cubic configurations form a complete framework [14]. In this paper, the invariant relationship for the twisted cubic configuration is applied to detecting critical configuration for camera calibration.

Camera calibration is a key problem for 3D reconstruction in the field of computer vision. One of the popular methods for this problem is to recover the camera parameters from the correspondences between image points and space points with known coordinates [1]. By using this method, many degenerate configurations may occur. There are systematic analyses for these degenerate configurations in Chapter 21 of [6], which consist of two cases: incidence case and non-incidence case. The incidence case is that some of the space points are collinear or coplanar, or some of the space points and the camera optical center are collinear or coplanar. The non-incidence case is that the space points and the camera optical center lie on a proper twisted cubic, of which no three points are collinear and no four points are coplanar [3]. The configuration of this non-incidence case is called twisted cubic degenerate configuration.

Detecting the degenerate configurations is important because the points of the degenerate configuration for camera calibration is critical and can result in dangerous recovered camera parameters. How to detect these degenerate configurations? For the incidence case, it is easy to detect by determining the linearly dependent relations among the space points or the image points. However, for the non-incidence case, it is difficult to detect. By the proof for Result 21.6 in Chapter 21 of [6], we think to solve camera projective matrix may be one of the methods to detect this non-incidence case, but a small noise may make failure. Similarly due to noise, our previous method [13] by the invariant representation of a twisted cubic is not practical. Moreover, recovering the optical center is necessary in this method.

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The established invariant relationships for a quadric cone configuration and a twisted cubic configuration in this paper are free of the camera optical center and camera projective matrix. They are used to construct a criterion function to detect the twisted cubic degenerate configuration. Simulations and experiments on real data performed show the proposed algorithm is reasonably stable against noise and the criterion function is quite useful in practice.

The organization of the paper is as follows. Some preliminaries are listed in Section 2. Section 3 gives the invariant relationships for a quadric cone or a twisted cubic configuration, and provides the algorithm to detect the twisted cubic degenerate configuration. Experiments are then followed in Section 4. And conclusion is made in Section 5.

2. Preliminaries

In this paper, a bold capital letter denotes either a homogeneous 4-vector or a matrix, a bold small letter denotes a homogeneous 3-vector, the bracket '[ ]' denotes the determinant of vectors in it. We assume that the camera optical center is non-collinear with two space points (i.e. different space points have different image points), no three image points are collinear and no four space points are coplanar (so the brackets on the image or space points are always non-zero). These assumptions are denoted as *ASS*.

Under the pinhole camera, a point $\mathbf{M}_i$ in space is projected to a point $\mathbf{m}_i$ in the image plane by

$$x_i\mathbf{m}_i = \mathbf{K}(\mathbf{R},\mathbf{t})\mathbf{M}_i, \quad i = 1\ldots6, \tag{1}$$

where $\mathbf{K}$ is the 3×3 matrix of camera intrinsic parameters, x_i is a non-zero scalar, and $\mathbf{R}$, $\mathbf{t}$ are a 3×3 rotation matrix and a 3D translation vector. The non-homogeneous coordinate of the camera optical center $\mathbf{O}$ not at infinity is $\hat{\mathbf{O}} = -\mathbf{R}^{\mathrm{T}}\mathbf{t}$. We assume that $\mathbf{O}$ is not at infinity throughout this paper.

Considering points $\mathbf{M}_i = (\hat{\mathbf{M}}_i^{\mathrm{T}},1)^{\mathrm{T}}, \quad i = 1\ldots6$, where $\hat{\mathbf{M}}_i$ are non-homogeneous coordinates, then we have (or see [4]):

$$\begin{aligned} x_ix_jx_k[\mathbf{m}_i,\mathbf{m}_j,\mathbf{m}_k] &= [x_i\mathbf{m}_i,x_j\mathbf{m}_j,x_k\mathbf{m}_k] \\ &= \det(\mathbf{K})[\mathbf{R}\hat{\mathbf{M}}_i + \mathbf{t},\mathbf{R}\hat{\mathbf{M}}_j + \mathbf{t},\mathbf{R}\hat{\mathbf{M}}_k + \mathbf{t}] \\ &= \det(\mathbf{K})[\hat{\mathbf{M}}_i + \mathbf{R}^{\tau}\mathbf{t},\hat{\mathbf{M}}_j + \mathbf{R}^{\tau}\mathbf{t},\hat{\mathbf{M}}_k + \mathbf{R}^{\tau}\mathbf{t}] \\ &= \det(\mathbf{K})[\hat{\mathbf{M}}_i - \hat{\mathbf{O}},\hat{\mathbf{M}}_j - \hat{\mathbf{O}},\hat{\mathbf{M}}_k - \hat{\mathbf{O}}] \\ &= \det(\mathbf{K})\begin{bmatrix} \hat{\mathbf{M}}_i, & \hat{\mathbf{M}}_j, & \hat{\mathbf{M}}_k, & \hat{\mathbf{O}} \\ 1, & 1, & 1, & 1, \end{bmatrix} \\ &= \det(\mathbf{K})[\mathbf{M}_i,\mathbf{M}_j,\mathbf{M}_k,\mathbf{O}], \quad i,j,k = 1\ldots6. \end{aligned} \tag{2}$$

Thus, $[\mathbf{m}_i, \mathbf{m}_j, \mathbf{m}_k] = 0$ if and only if $[\mathbf{M}_i, \mathbf{M}_j, \mathbf{M}_k, \mathbf{O}] = 0$, which means $\mathbf{m}_i$, $\mathbf{m}_j$, $\mathbf{m}_k$ are collinear if and only if $\mathbf{M}_i$, $\mathbf{M}_j$, $\mathbf{M}_k$, $\mathbf{O}$ are coplanar.

Hereafter, for the notational convenience, if no ambiguity can be aroused, $\mathbf{M}_i$, $i = 1\ldots6$ will be simply denoted as **1**, **2**, **3**, **4**, **5**, **6**, and the commas in brackets will be omitted.

Theorem 1 in [13] is recalled here. For six points **1**, **2**, **3**, **4**, **5**, **6** with no three collinear and no four coplanar, there is a unique proper quadric cone with **1** as the vertex and through **2**, **3**, **4**, **5**, **6**. Any point **X** is on the quadric cone if and only if:

$$\frac{[\mathbf{1234}][\mathbf{1356}]}{[\mathbf{1236}][\mathbf{1456}]} = \frac{[\mathbf{124X}][\mathbf{135X}]}{[\mathbf{123X}][\mathbf{145X}]}. \tag{3}$$

This representation is not unique as a result that the one after a permutation of **2**, **3**, **4**, **5**, **6** is also a representation of the same quadric cone.

Theorem 2 in [13] is also recalled here. For six points **1**, **2**, **3**, **4**, **5**, **6** with no three collinear and no four coplanar, there is a unique proper twisted cubic passing through them. Any point **X** is on the twisted cubic if and only if:

$$\begin{cases} \dfrac{[\mathbf{1246}][\mathbf{1356}]}{[\mathbf{1236}][\mathbf{1456}]} = \dfrac{[\mathbf{124X}][\mathbf{135X}]}{[\mathbf{123X}][\mathbf{145X}]} \\ \dfrac{[\mathbf{1246}][\mathbf{2356}]}{[\mathbf{1236}][\mathbf{2456}]} = \dfrac{[\mathbf{124X}][\mathbf{235X}]}{[\mathbf{123X}][\mathbf{245X}]}, \end{cases} \quad \text{and } \mathbf{X} \text{ is not on the lie } \mathbf{12}. \tag{4}$$

This representation is not unique as a result that the one after a permutation of **1**, **2**, **3**, **4**, **5**, **6** is also a representation of the same twisted cubic.

Let the projection matrix $\mathbf{K}(\mathbf{R},\mathbf{t})$ in (1) be $\mathbf{P}$. The direct-linear-transformation (DLT) method [1], a popularly used camera calibration method, is to solve $\mathbf{P}$ linearly from at least six pairs of the space points and their image points and then to decompose $\mathbf{P}$ as $\mathbf{K}$, $\mathbf{R}$, $\mathbf{t}$. When the space points and the camera optical center lie on a twisted cubic, no matter how many pairs of the space and image points there are, $\mathbf{P}$ cannot be determined uniquely. This twisted cubic configuration is the twisted cubic degenerate configuration mentioned in Section 1.

3. Detecting twisted cubic degenerate configuration

3.1. Invariant relationships for a quadric cone configuration or a twisted cubic configuration

Proposition 1. *There is a unique quadric cone with* 1 *as the vertex and passing through* **2**, **3**, **4**, **5**, **6**. *The camera optical center* **O** *lies on this quadric cone if and only if*:

$$\frac{[\mathbf{1246}][\mathbf{1356}]}{[\mathbf{1236}][\mathbf{1456}]} - \frac{[\mathbf{m}_1\mathbf{m}_2\mathbf{m}_4][\mathbf{m}_1\mathbf{m}_3\mathbf{m}_5]}{[\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3][\mathbf{m}_1\mathbf{m}_4\mathbf{m}_5]} = 0. \tag{5}$$

Each ratio in (5) *is a cross ratio. After a permutation of* **2**, **3**, **4**, **5**, **6** *and their corresponding image points in* (5), *the obtained equation is still an invariant relationship of* **O** *lying on the same quadric cone*, *but is not independent of* (5).

Proposition 1 is proved as follows: according to the recalled Theorem 1 of [13] in Section 2, there is a unique proper quadric cone with 1 as the vertex and through **2**, **3**, **4**, **5**, **6**. By (3) and (2), **O** lies on the quadric cone if and only if:

$$\frac{[\mathbf{1246}][\mathbf{1356}]}{[\mathbf{1236}][\mathbf{1456}]} = \frac{[\mathbf{124O}][\mathbf{135O}]}{[\mathbf{123O}][\mathbf{145O}]} = \frac{[\mathbf{m}_1\mathbf{m}_2\mathbf{m}_4][\mathbf{m}_1\mathbf{m}_3\mathbf{m}_5]}{[\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3][\mathbf{m}_1\mathbf{m}_4\mathbf{m}_5]}.$$

Since, in (3) we can permute **2**, **3**, **4**, **5**, **6**, in this equation we also can permute the points and their corresponding image points.

Similarly, according to (4) and (2), we have:

Proposition 2. *The camera optical center* **O** *lies on the proper twisted cubic passing through* **1**, **2**, **3**, **4**, **5**, **6** *if and only if*

$$\begin{cases} \dfrac{[\mathbf{1246}][\mathbf{1356}]}{[\mathbf{1236}][\mathbf{1456}]} - \dfrac{[\mathbf{m}_1\mathbf{m}_2\mathbf{m}_4][\mathbf{m}_1\mathbf{m}_3\mathbf{m}_5]}{[\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3][\mathbf{m}_1\mathbf{m}_4\mathbf{m}_5]} = 0, \\ \dfrac{[\mathbf{1246}][\mathbf{2356}]}{[\mathbf{1236}][\mathbf{2456}]} - \dfrac{[\mathbf{m}_1\mathbf{m}_2\mathbf{m}_4][\mathbf{m}_1\mathbf{m}_3\mathbf{m}_5]}{[\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3][\mathbf{m}_1\mathbf{m}_4\mathbf{m}_5]} = 0. \end{cases} \tag{6}$$

Each ratio in (6) *is a cross ratio. After a permutation of* **1**, **2**, **3**, **4**, **5**, **6** *and the corresponding image points, the obtained equation system is still the invariant relationship of* **O** *lying on the same twisted cubic, but is not independent of* (6).

Note that in (4), there is another condition for the representation of the twisted cubic: **O** does not lie on the line through **1** and **2**. Here, this additional condition is unnecessary because if **O** is on the line through **1** and **2**, then $\mathbf{m}_1 = \mathbf{m}_2$, which is contrary to our assumption *ASS*.

The image points are con-conic when the camera optical center and space points lie on either a quadric cone or a twisted cubic. It is not surprising that the image of a twisted cubic containing the optical center is also a conic, because a twisted cubic can be generated as the intersection of two quadric cones, and the projection of a twisted cubic from any a point on it to a plane is a planar conic [12]. The difference for the quadric cone and twisted cubic cases is that the numbers of the equations in (5) and (6) are different, and furthermore the quadric cone configuration is not degenerate for camera calibration but the twisted cubic configuration is [14]. Notice that the image points are con-conic is only a necessary condition for the twisted cubic degenerate configuration. Therefore, it is sometimes not reliable to detect the degenerate configuration by detecting whether the image points are con-conic or not.

We have established the above invariant relationships between space and image points for a quadric cone or a twisted cubic configuration. The previous work [2,4,5,7–11] and them form a complete framework for the invariance of six points. Interested readers can find the details in our paper [14].

3.2. Implementation

We are to apply the invariant relationships of Section 3.1 to recognize the twisted cubic degenerate configuration for camera calibration. Other degenerate configurations are in the incidence case that can be detected easily by determining the linearly dependent relations among the space or image points.

In the invariant relationship (6), $\mathbf{m}_6$ does not occur. Moreover, by Proposition 1, we know that the first equation in (6) is the cone with **1** as the vertex, the second equation in (6) is the cone with **2** as the vertex. Thus, the two equations in (6) are denoted as $g_{\mathbf{1,24,35}}(\mathbf{6})=0$, $g_{\mathbf{2,14,35}}(\mathbf{6})=0$. Stability to noise of these two equations is much affected by the order of space and image points. So, we are to consider more equations after changing their orders. We do a permutation on **1**, **2**, **3**, **4**, **5**, **6** and their corresponding images in $g_{\mathbf{1,24,35}}(\mathbf{6})=0$, and denote the result as $g_{\mathbf{i,jk,pq}}(\mathbf{M})=0$, where **i**, **j**, **k**, **p**, **q**, **M** is a permutation on **1**, **2**, **3**, **4**, **5**, **6**. For a fixed **M**, there are in total 30 different such equations when varying the orders of **i**, **j**, **k**, **p**, **q**.

An algorithm to detect whether at least six space points and camera optical center lie on a twisted cubic degenerate configuration is outlined as follows:

Step 1. Make a set *ST*, of which each element is a six-pair group of space and image points. The six-pair group satisfies *ASS*, of which five pairs distributing uniformly are denoted as **1**, **2**, **3**, **4**, **5**, $\mathbf{m}_i$, $i=1\ldots5$ and the rest pair is denoted as **M**, **m**. Each pair of space and image points appears in *ST*.

Step 2. For each six-pair group in *ST*, set up all of the 30 equations $g_{\mathbf{i,jk,pq}}(\mathbf{M})$ varying the orders of **1**, **2**, **3**, **4**, **5**, where $\{\mathbf{gi}, \mathbf{jk}, \mathbf{pq}(\mathbf{M})\} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}\}$. And, let the subscript set $g_{\mathbf{i,jk,pq}}(\mathbf{M})=0$ be *S*. Determine whether $\frac{1}{30}\sum_{(\mathbf{i,jk,pq})\in S}|g_{\mathbf{i,jk,pq}}(\mathbf{M})| < \varepsilon$, where ε is a preset threshold. If yes for all groups in *ST*, then the space points and the camera optical center lie on a proper twisted cubic. Otherwise, they are not on a twisted cubic.

Denote the criterion function $\frac{1}{30}\sum_{(\mathbf{i,jk,pq})\in S_1}|g_{\mathbf{i,jk,pq}}(\mathbf{M})|$ as $f(\mathbf{M})$.

Remark. In the above Step 1, if we cannot obtain the set *ST* containing all the pairs of space and image points, the used space configuration is special. Usually, this configuration will not be chosen to calibrate camera by the DLT method.

4. Experiments

4.1. Simulations

Many experiments show the proposed algorithm is stable to noise. Three samples are reported below.

The world coordinate system is taken as the camera coordinate system. The simulated camera intrinsic parameters are

$$\mathbf{K} = \begin{pmatrix} 1000 & 0 & 512 \\ 0 & 900 & 384 \\ 0 & 0 & 1 \end{pmatrix},$$

then the images of seven space points **1**, **2**, **3**, **4**, **5**, **6**, **7** are generated, where **1**, **2**, **3**, **4**, **5**, **6**, **O** do not lie on a twisted cubic, and **1**, **2**, **3**, **4**, **5**, **7**, **O** do lie on a twisted cubic. The used three images are shown in Fig. 1 and their image sizes are not greater

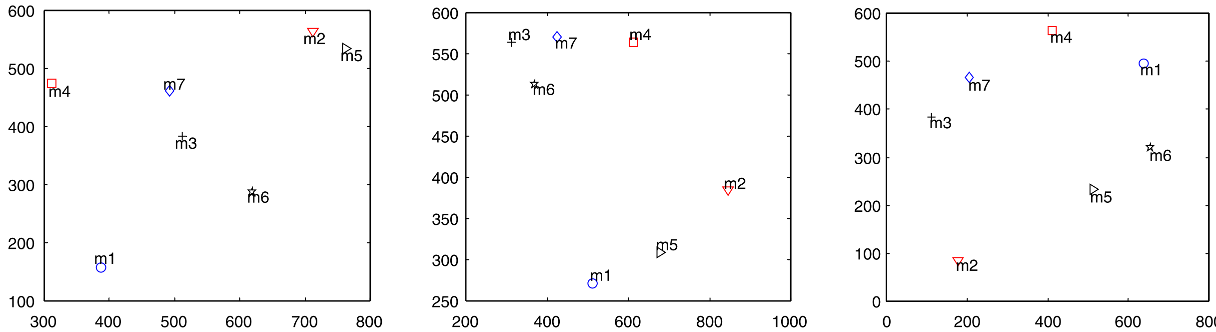

Fig. 1. Three simulated images, denoted as D_i, $i = 1 \ldots 3$.

than 800×600 pixels. Gaussian noise with mean 0 and standard deviation ranging from 0 to 6 pixels is added to each image points, then the values of $f(\mathbf{6})$, $f(\mathbf{7})$ are computed. For each noise level, we perform 100 runs, and the calculated averaged results, still denoted by $f(\mathbf{6})$, $f(\mathbf{7})$, are shown in Table 1. Since, **1**, **2**, **3**, **4**, **5**, **7**, **O** lie on a twisted cubic and **1**, **2**, **3**, **4**, **5**, **6**, **O** do not lie on a twisted cubic, $f(\mathbf{7})$ should be close to zero, while $f(\mathbf{6})$ should not. Therefore, there should be $f(\mathbf{6}) > f(\mathbf{7})$. The results in Table 1 show all the values of $f(\mathbf{7})$ are small and all the values of $f(\mathbf{6})$ are not. Simultaneously, the results show the criterion function f is quite stable to noise.

The stability of $f(\mathbf{7})$ when **7** moves on a fixed twisted cubic is also tested. Extensive simulations show $f(\mathbf{7})$ does not depend on the position of **7** very much. A sample is shown as follows. We take one fixed group of **1**, **2**, **3**, **4**, **5**, **O** and their images. Then, the parametric equations of the twisted cubic through **1**, **2**, **3**, **4**, **5**, **O** are generated by the method in [13]. Taking different parameters 3, 4, 9/2, 5, 6, −5, we

Table 1
The values of $f(\mathbf{6})$, $f(\mathbf{7})$ under different noise levels

Noise level (pixel)		0	1	2	4	6
D_1	$f(\mathbf{6})$	7.40	7.40	7.41	7.41	7.45
	$f(\mathbf{7})$	0.00	0.04	0.09	0.18	0.31
D_2	$f(\mathbf{6})$	3.69	3.66	3.73	4.00	5.02
	$f(\mathbf{7})$	0.00	0.13	0.26	0.70	1.66
D_3	$f(\mathbf{6})$	2.62	2.62	2.62	2.62	2.62
	$f(\mathbf{7})$	0.00	0.01	0.02	0.04	0.06

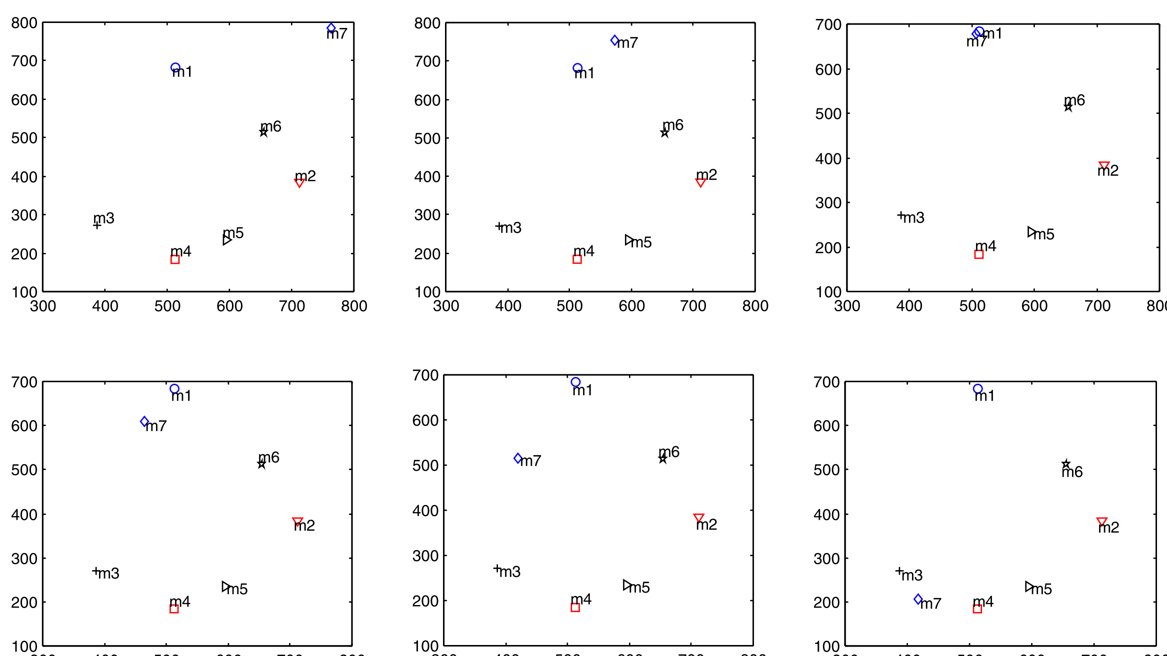

Fig. 2. The images of **1**, **2**, **3**, **4**, **5**, **7** with **7** moving on the twisted cubic of **1**, **2**, **3**, **4**, **5**, **O**, where the parameters of **7** are 3, 4, 9/2, 5, 6, −5, respectively.

Table 2
The values of $f(\mathbf{7})$ with **7** moving under different noises

Par.	Noise				
	0	1	2	4	6
3	0.00	0.04	0.10	0.17	0.29
4	0.00	0.04	0.08	0.15	0.23
9/2	0.00	0.04	0.07	0.19	0.27
5	0.00	0.05	0.08	0.19	0.26
6	0.00	0.04	0.07	0.17	0.28
−5	0.00	0.04	0.08	0.18	0.29

obtain different space point **7**. The different images of **1**, **2**, **3**, **4**, **5**, **7** with **7** varying are shown in Fig. 2. At each position of **7** and at each noise level, the averaged result of 100 runs of $f(\mathbf{7})$ is computed and shown in Table 2. We can see that $f(\mathbf{7})$ has high stability against noise.

4.2. Experiments on real data

A real image of a calibration grid taken by a NIKON COOLPIX990 camera is shown in Fig. 3. The size of this image is 1024×768 pixels. We extract the pixels of the edges by Canny edge detector, then fit them as lines, and calculate the intersection points of these lines. The world coordinate system is set up in the grid. We obtain 108 pairs of space and image points. By using DLT method [1] from these 108 pairs of space and image points, we compute the camera intrinsic parameter matrix **K**, and the camera pose parameters: rotation **R** and translation **t**. The results are:

$$\mathbf{K} = \begin{pmatrix} 2049.8128, & -2.7983, & 523.9202 \\ 0, & 2050.5605, & 294.1385 \\ 0, & 0, & 1 \end{pmatrix},$$

$$\mathbf{R} = \begin{pmatrix} 0.7784, & -0.6272, & 0.0270 \\ -0.2648, & -0.3671, & -0.8917 \\ 0.5692, & 0.6870, & -0.4518 \end{pmatrix},$$

$$\mathbf{t} = \begin{pmatrix} -0.7503 \\ 4.8624 \\ 30.7296 \end{pmatrix}.$$

Fig. 3. A real image of a calibration grid.

We randomly combine 126 six-pair groups with no three image points collinear, no four space points coplanar. We choose the group with the maximal value of $f(\mathbf{6})$ and the group with the minimal value of $f(\mathbf{6})$. The two values are 21.5821 and 0.2154. The group with value 21.5821 is denoted as $G_{\max}$, and the group with value 0.2154 is denoted as $G_{\min}$. We calibrate the camera from $G_{\max}$ and the results are:

$$\mathbf{K}_1 = \begin{pmatrix} 2101.9090, & -2.3031, & 501.8081 \\ 0, & 2101.4320, & 325.7270 \\ 0, & 0, & 1 \end{pmatrix},$$

$$\mathbf{R}_1 = \begin{pmatrix} 0.7857, & -0.6319, & 0.0279 \\ -0.2663, & -0.3717, & -0.8989 \\ 0.5593, & 0.6936, & -0.4166 \end{pmatrix},$$

$$\mathbf{t}_1 = \begin{pmatrix} -0.7618 \\ 4.9034 \\ 30.8571 \end{pmatrix}.$$

Similarly, we calibrate the camera from $G_{\min}$ and the results are:

$$\mathbf{K}_2 = \begin{pmatrix} 980.4078, & 26.8782, & 430.9372 \\ 0, & 870.5113, & 541.8497 \\ 0, & 0, & 1 \end{pmatrix},$$

$$\mathbf{R}_2 = \begin{pmatrix} -0.6666, & -0.7454, & 0.0062 \\ -0.0205, & -0.0266, & 0.9994 \\ 0.7451, & 0.6661, & 0.0330 \end{pmatrix},$$

$$\mathbf{t}_2 = \begin{pmatrix} -1.0456 \\ -1.5375 \\ -18.0317 \end{pmatrix}.$$

We evaluate $\mathbf{K}_1$, $\mathbf{R}_1$, $\mathbf{t}_1$, $\mathbf{K}_2$, $\mathbf{R}_2$, $\mathbf{t}_2$ by comparing them with **K**, **R**, **t**:

$$\mathbf{K}-\mathbf{K}_1 = \begin{pmatrix} -52.0962, & -0.4952, & 22.1121 \\ 0, & -50.8715, & 68.4115 \\ 0, & 0, & 0 \end{pmatrix},$$

$$\mathbf{K}-\mathbf{K}_2 = \begin{pmatrix} 1069.4050, & -29.6764, & 92.9830 \\ 0, & 1180.0491, & -147.7112 \\ 0, & 0, & 0 \end{pmatrix},$$

$$\mathbf{R}-\mathbf{R}_1 = \begin{pmatrix} -0.0066, & -0.0082, & 0.0036 \\ -0.0192, & -0.0213, & 0.0139 \\ 0.0005, & -0.0182, & -0.0283 \end{pmatrix},$$

$$\mathbf{R}-\mathbf{R}_2 = \begin{pmatrix} 1.4450, & -1.3725, & 0.0208 \\ -0.2443, & -0.3405, & -1.8911 \\ -0.1759, & 0.0208, & -0.4848 \end{pmatrix},$$

$$\mathbf{t}-\mathbf{t}_1 = \begin{pmatrix} -0.3248 \\ -1.0234 \\ -0.6420 \end{pmatrix}, \qquad \mathbf{t}-\mathbf{t}_2 = \begin{pmatrix} 0.2954 \\ 6.3999 \\ 48.7613 \end{pmatrix}.$$

It is seen that the estimations from $G_{\max}$ are much better than those from $G_{\min}$.

Therefore, we see that the calibration result from six pairs of space and image points with smaller value of the criterion function f (i.e. space points and optical center are near to the twisted cubic degenerate configuration) is not better than the one with larger value of the criterion function f (i.e. space points and optical center are far from the twisted cubic degenerate configuration). The proposed criterion function, thus, can be trusted for camera parameter estimation.

5. Conclusions

We have presented the invariant relationships between six space points and their image points when camera optical center and the space points lie on a quadric cone or twisted cubic. Then, the invariant relationship for the twisted cubic configuration is used to recognize the non-trivial degenerate configuration of more than six points for camera calibration by an algorithm, where no explicit computation on the optical center or projective matrix is needed. We believe that the invariant relationships have further usefulness. For example, when applying a RANSAC during the process of determining camera parameters, using the invariant relationships can filter out the initial critical groups.

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