Reconstruction of Tree Crown Shape from Scanned Data

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Abstract. Reconstruction of a real tree from scattered scanned points is a new challenge in virtual reality. Although many progresses are made on main branch structures and overall shape of a tree, reconstructions are still not satisfactory in terms of silhouette and details. We do think that 3D reconstruction of the tree crown shapes may help to constrain accurate reconstruction of complete real tree geometry. We propose here a novel approach for tree crown reconstruction based on an improvement of alpha shape modeling, where the data are points unevenly distributed in a volume rather than on a surface only. The result is an extracted silhouette mesh model, a concave closure of the input data. We suggest an appropriate scope of proper alpha values, so that the reconstruction of the silhouette mesh is a valid manifold surface. Experimental results show that our technique works well in extracting the crown shapes of real trees.

Keywords: tree crowns, reconstruction, Delaunay triangulation, alpha shape.

1 Introduction

With the current development of virtual environment establishment, product design, digital entertainment, antique protection, and city programming 3D geometry model construction and processing is now an active development area. 3D geometry modeling is regarded as the fourth digital multimedia in addition to digital audio, digital image, and digital video. 3D geometry models are normally used to represent object surface to identify extendedly shape and appearance attributes.

With the advancement of 3D scanning technology, more and more 3D digital scanners are popularly used for different applications. Rich details of the object shape can be acquired from scanned data with dense sampling points (point cloud), where no topological connection relations are included. It becomes important to develop new processing methods to represent, to process, to reconstruct and to render these highly complex geometric bodies. Reconstruction of geometry model is one of the important research topics in modern virtual reality.

Trees are typical objects in virtual reality, so it is very important to reconstruct and to represent the real trees. Tree reconstruction can be used in many applications, including digitization of vegetation scenes, design of a new scene, digital entertainment, and so on.

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Reconstruction of the tree crown shape is useful to model a real tree and in various research fields interested by growth simulation of virtual trees for light interception, biomass evaluation, and so on. Many researches have been carried on surface reconstructions, but the shape of a tree crown is more difficult than that of a usual solid object in its heavy occlusions and its high complexity in geometry and topology.

Reconstruction of a real tree crown from scattered scanned points is a new challenge in virtual reality. The points are unstructured, unevenly distributed, and sampled from non-manifold shapes; it is very difficult to define typical boundary points in a tree crown in accordance with the visual perception. Rich concavity is another feature of the crown shape of a real tree. Classical surface reconstruction techniques do not work for such tree crown data. Other difficulties are that the data have no topological information among points, their 3D distribution is not even at all, and they are not complete; so it becomes rather difficult to reconstruct it with sufficient details.

Alpha shape is a new technique [1] in the classification of all the simplexes from 3D Delaunay triangulation of a 3D point set, and the result of this classification is three categories: the internal simplexes of the shape, the regular one and the external one. With a proper heuristic alpha value specified by the user [1], a concave silhouette shape of a point set sampled from a regular manifold surface can be constructed.

We will improve this approach to the point cloud data acquired from the scan on a real tree. Because of the limitations of alpha shape technique and complexity of scanned tree data, it is not possible to have all details of the tree crown reconstructed to a regular mesh through a direct application of the alpha shape technique [1]. In this paper we solve this problem using a range of alpha values and testing the close property of the constructed mesh, so that the mesh model is a concave closure of the point data.

The structure of this paper is as follows. Related work in shape information analysis and plant modeling is introduced in section 2. Fundamental knowledge of our method is described in section 3. Technical details of this new approach are described in section 4. Experiments of this technique to reconstruct tree crown shapes are shown in section 5. Conclusions about this technique and further investigation are described in section 6.

2 Related Work

In the past decades, many methods have been developed on point shape processing and shape modeling of complex objects including plants, but with unequal results.

2.1 Point Geometry Processing

Jarvis [2] was the first to consider the problem of computing the shape as a generalization of the convex hull of a planar point set. A mathematically definition of the shape was developed by Edelsbrunner in [3]. For 3D points, Boissonnat [4] suggested to use Delaunay triangulation to “sculpture” a single connected shape of a point set.

In the frame of projects such as the digital Michelangelo project [5] at Stanford Computer Graphics Lab in the 2000’s, and with the improvement of computer hardware, a numerous number of research papers have been published on point cloud processing and rendering. Point geometry processing and analysis became an active research topic.

2.2 Plant Modeling on Knowledge and Rules

The different approaches of 3D tree model construction can be roughly classified into three categories: botanical models, geometrical models, and digitized models from real plants.

There are a numerous methods to simulate real plant appearance. Many early methods were based on rule iterations (botanical, physical, geometrical, mathematical), or simply based on strong user control with advanced dedicated patterns. In the 1980s, modeling by botanical rules appeared, and produced nice findings, researchers tried to simulate the growth of natural plants, plants could be constructed by some botanical rules or grammars. AMAP [6] modeling method is based on bud life cycles of botanical knowledge with real measurement data (on plant topology). This modeling method clear reflects the growth mechanism of plants, including space occupation and the location of leaves, fruits, and flowers. L-systems presented by a Lindenmayer and Prusinkiewicz were broadly applied to describe the growth process of plant organs, which were based on fractal pattern [7, 12].

GreenLab [13] modeling approach is put forward as a mathematical model, which simulates interactions of plant structure, leaves, trunk, branch and function. This model can exactly engender the dynamics of plant, architecture and geometry of woody plants, because of internal competition for resources, leaves sizes are different, and growth of pruning can also be simulated.

These methods, used mainly in biology research fields are not dedicated to control the 3D plant shape, and cannot easily do it, but aim to understand plant shape as the result of a dynamic. It as be cited that this kind of model is not suitable to construct a 3D models of real tree by using botanical methods [10].

Geometrically interactive modeling is another way to model virtual plants. Although this method does not strictly follow the botanical rules, but visually realistic trees can be produced [14]. In general, given 3D skeleton points of real plant, 3D model of each branch can be generated with generalized circular cylinders [15]. Prism model is a simplified application of this method. This approach is widely applied in some plant software such as Xfrog, if combining rule-based method with traditional geometric modeling approach. Nice 3D plant model could be produced, such as flowers, bushes, and trees [8, 14].

To summarize, these rule based or pattern based methods used to build the real plant faithful to botanic knowledge or appearance, can produce visually very realistic plants, although they could not be used to model a specific existing real plant.

2.3 Digitalization of Real Plants

New modeling methods have been developed to digitalize real plants in very recent years [9-11]. These methods can be used to reconstruct the trunk, the branches, and the leaves, but the realism of the reconstructed model is still different from the real shape due to the lack of crown silhouette shape information.

Plant digitization aims to reconstruct the shape of real plants from the information digital instruments. The most popular techniques are the use of 3D laser scanner [9] or the use of digital photos [10].

When scanning a real plant from a single viewpoint many occlusions occur. In particular, many leaves do usually hide branches and other organs from the view. One way to make reconstruction efficient is to work both on plant branching structure reconstruction and on plant crown reconstruction. The idea of the proposed approach is to consider that when processing the branch reconstruction of a real plant, we must constrain the silhouette of branches from the crown shape.

By scanning a real tree, we have a point cloud data set, from which we could reconstruct the shape of the real tree crown by combining existing methods.

Considering the branch structure, we may underline several interesting works. Cheng [16] reconstructs a real tree from a range image, using generalized circular cylinders to fit incomplete data and compute the skeleton based on axis direction. Pfeifer [17] introduces a set of algorithms for automatically fitting and tracking cylinders along branches and reconstructing the entire tree.

With the appearance of advanced precise digital camera and laser scanner, the development of digital plant is accelerated. Image-based and laser-scanning based methods have come up to produce 3D model of real trees in nature. Shlyakhter [18] builds a 3D model of tree from a set of photographs. His method constructs the visual hull of tree first, then a plausible skeleton is built up from medial axis of visual hull, and finally L-system is applied to construct branches and leaves. Teng [19] reconstruct 3D trunk of plant only from two images, this method only estimates skeleton and radius of branches roughly. Quan [10] also models a plant from digital image. Their work focus on reconstruction of big leaves, branches are reconstructed by interaction.

These image-based approaches can build 3D plant from images of different viewpoints, but because of inevitable noise of images and error of camera parameters, the accuracy of those methods is limited.

The approaches of Xu [9, 11] are based on some prior knowledge. A skeleton is first constructed by connection of the centroids of points, which have an analogous length of the shortest path to a root point. Then the corresponding radius of skeleton nodes could be computed by the allometric theory. Leaves are constructed in the end, so that the reconstructed tree is visually impressive.

However during the reconstruction, the methods of imaged-based or 3D laser scanning data based first construct the skeleton, and then construct leaves, but because of much occlusion, the reconstructed skeleton is incomplete. 3D laser scanner could not scan the thin branches because of its limited precision.

But we have to reconstruct these thin branches for the architecture shape of real plants in botany and digital forestry and for high visual impression in virtual reality.

We must thus construct the shape of tree crown to constrain the reconstruction of thin branches.

3 Algorithm Bases: Alpha Shape

Alpha shape was proposed in 2D by Edelsbrunner [3], and was then extended to 3D in [1]. This method can be used to reconstruct object surface from an unorganized point cloud. Our reconstruction of concave tree crown is based on this technique.

3.1 Delaunay Triangulation

A set P of points can be used to construct a complex if the points do not lay in a plane. Delaunay triangulation is a natural choice to do it. In literature, different Delaunay triangulation techniques are proposed [20-22], where Lawson flip method is a typical one. in Lawson's method, the tetrahedron bounding the point set P is constructed at first, and the other points are inserted into the triangulation one by one then. Each time, the triangulation is optimized to satisfy the Delaunay property: the circumsphere of every tetrahedron does not contain any other points. Those tetrahedrons, which do not satisfy a local Delaunay property, are flipped.

The flip process in 3D can be described as follows. The triangulation in 3D is a set of tetrahedrons constructing a simplical complex. We will explain the case of two tetrahedrons incident to a triangle ace (Figure 1). If the circumsphere of tetrahedron $aecd$ does not contain b and circumsphere of tetrahedron $aecb$ does not contain d, it can be said that (the triangle)$\square aec$ is local Delaunay. Otherwise, this situation can be modified inserting a new edge bd inserted. Therefore the complex is a Delaunay triangulation.

The result of Delaunay triangulation of the point set is its convex hull composing several tetrahedrons.

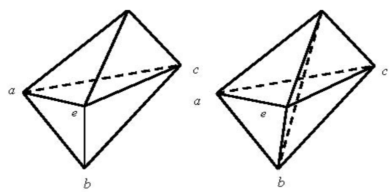


Fig. 1. Flipping in three dimensions

3.2 Alpha Shape

The concept of alpha-shapes formalizes the intuitive notion of *shape* for spatial point sets on user's selection. Alpha-shape is a mathematically well-defined generalization of the convex hull. Its result is a series of subgraphs of the Delaunay triangulation, depending on different alpha values. Given a finite point set, a family of simplexes can be computed from the Delaunay triangulation of the point set, a real parameter alpha controls the desired level of detail. All real alpha values lead to a whole family of shapes. The alpha-shape of a point set is made up of the set of points, edges, triangles and tetrahedrons, which satisfy the constraint condition: the alpha test [1]. This test applies for each a triangle t of the triangulation. If t is not on the boundary of the convex hull, there must be two tetrahedrons p,q , which are incident to t . Tetrahedrons p and q are tested to be in the circumsphere of t or not. If they both are not in that circumsphere, and the radius of the circumsphere is less than the alpha value, t is said to satisfy alpha test, and it is regarded as one member of the alpha shape. So alpha-shape is a subset of the triangulation.

If we let alpha be large enough, the shape is the convex hull of the points set. If alpha approaches 0, no tetrahedral, triangles and edges could pass the alpha test, so the alpha shape is the points set. With the adjustment of the alpha values, this subset can follow the topology of the points set. So, if we choose a proper value for alpha, we will find a reasonable surface for a tree crown.

The alpha shape is a sub-complex of the Delaunay triangulation of the points set P. This can be explained in the following. There is a ball eraser with alpha as its radius, and it could move to all possible positions in the 3D space and with no point of P included. This eraser will delete all simplexes whose size is bigger than alpha and it can pass through. So the remaining simplexes construct the alpha shape.

4 Shape Construction of Tree Crown

The most impressive aspect of a tree is the silhouette of its crown, so the shape of the crown surface is one important aspect for tree reconstruction for the virtual environments. We can only acquire discrete points of the crown with the most recent sensors in nowadays. Normally the data with 3D laser scanner are range images, each of which is obtained from the scan at single viewpoint.

Point cloud from leaves determines the shape of tree crown. Since branches support leaves in the architecture, branch reconstruction is important also. If we do not have the branch model, we do not know how to locate the leaves. Reconstruction of branches consistent to tree crown should be the main target of the reconstruction of a real tree. It is very hard to reconstruct tree branches directly since shape information of the point data is rather weak. The data for branches are incomplete due to the occlusion of leaves and other branches. On the other hand, some little twigs cannot be scanned because of precision limit laser spots.

If we build up the surface of a tree crown from the scanned data, the reconstruction of tree branches will be easier under the control of tree crown surface. Otherwise, the reconstruction result might be different from the real tree, so not faithful to be applied to tree measurement.

4.1 Analysis of Scanned Data of a Real Tree

It is an ordinary technique to sample the surface of real object using 3D laser scanner, and then to reconstruct the shape from the sample data with limited precision. This point cloud data describe the geometry and the appearance attribute of objects surface. The normal point cloud is densely sampled from continuous or smooth surface, although the data is unorganized and irregular. A number of successful methods have been presented to deal with these data and to reconstruct appearance of object.

Plants, such as trees, have too many organs and its structure is too complex. A tree is made up of trunk, branches, and a huge number leaves. The point cloud data of tree is not sampled from a manifold surface, so it is more irregular than those from other data from the manifold surface. The points from leaves are even more irregular. The density variation of point cloud from leaves may be very large. Thus, traditional technique does not work for these objects. Special methods should be developed to reconstruct real plants. In order to keep the shape of plants, branch skeleton extraction and construction of plant crown should be included. One difficulty of this work is that the points from branches and those from leaves are mixed together, so that it is hard to initialize the work of shape analysis.

4.2 Building the Mesh Model of Tree Crown

From the above analysis and the range image data acquired from a single scan in Figure 3(a) and Figure 4(a), we can recognize the dense region and the sparse region of the data by observation, but this recognition process is very difficulty to be performed in a computer. The points from the tree side facing the scanner and the region with dense leaves (the side of a tree facing the sun, for example) are denser. There may be some interstices among dense leaves. When we scan a tree, laser lights will pass the interstice and meet the branch or leaves at another side of tree, or pass through the tree. So there should be holes in the data. Although we can distinguish the dense region, the sparse region, the convex region and concave region, the algorithms processing very dense point set will make mistakes in topological reconstruction. Therefore, we must construct topological structure of points at first, where Delaunay triangulation is an ideal choice.

Our algorithm contains four steps:

The first step is to triangulate point set $P=\{p_i\}$ with Delaunay triangulation, so that a set of connected tetrahedrons $T=\{T_j\}$ are obtained. Flipping method in [23] is adopted to correct irregular triangulation in $P=\{p_i\}$. All tetrahedrons $T=\{T_j\}$ will constitute a convex solid, the shell of this solid is a convex hull.

The second step is to compute all radii, $R(T_j)$, of circumsphere of every tetrahedron after triangulation. This value will be one attribute of a tetrahedron T_j. The radii, $r(F_k)$, of the circumcircle of each face of a tetrahedron arecomputed also, and they are thought of as an attribute of each face.

The third step is to classify tetrahedrons $\{T_j\}$ and their all faces. The rule to classify all T_j is the size of $R(T_j)$. This classification is performed by the relation of $R(T_j)$ with threshold α and, where α is specified by users. The scope of α should be proper. Then all tetrahedrons are classified into two categories according to a real value α: interior tetrahedrons and exterior tetrahedrons. If $R(T_j)>\alpha$, T_j is classified as an exterior tetrahedron. Otherwise, it is classified as an interior tetrahedron. All faces $\{F_k\}$ from each tetrahedron T_j are classified into three categories also: interior faces, exterior faces and boundary faces. The classification role is as follows. If a face on the convex hull belongs to an exterior tetrahedron, it is an exterior faces; otherwise, if it belongs to an interior tetrahedron, it is a boundary faces. For each face not on the hull, if it is an intersection face of two exterior tetrahedrons, it is an exterior face. If it is an intersection face of two interior tetrahedrons, it is an interior face. If it is an intersection face of one interior tetrahedron and one exterior tetrahedron, it is a boundary face. All boundary faces will construct a mesh, and this mesh M will be an concave approximation of the crown.

Let rmax be the largest radius of all $R(T_j)$ and all $r(F_k)$, and Let rmin be the smallest radius of all $R(T_j)$ and all $r(F_k)$. We acquire an interval $[A,B]$, where $A=\lambda r\min$, $B=\mu r\max$, $\lambda=0.9$, and $\mu=1.1$. The α value should be confined to the

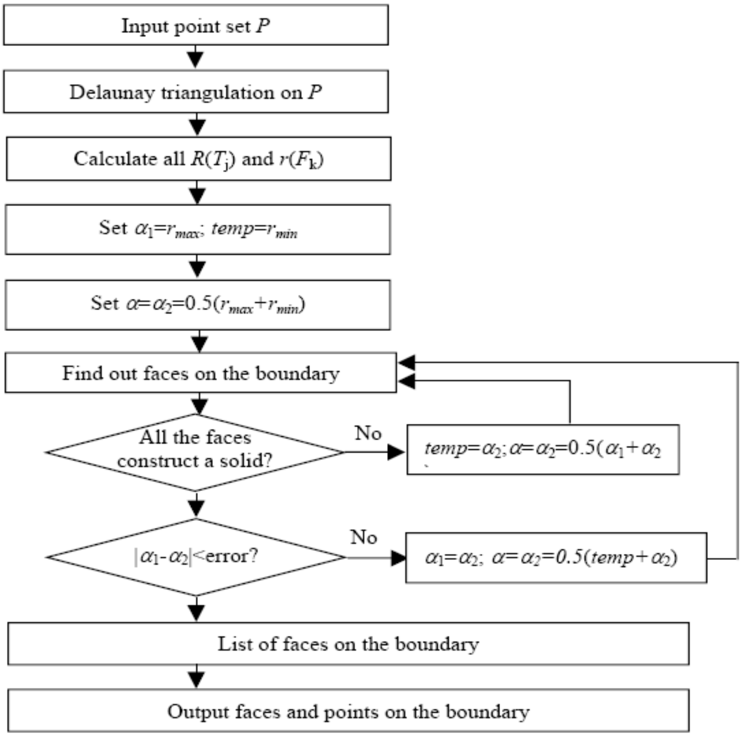


Fig. 2. Pipeline of this algorithm

interval $[A, B]$; otherwise, if $\alpha > B$, the mesh M will be a convex hull, and if $\alpha < A$, the mesh M will not be a solid.

The fourth step is to test the validity of specific alpha values, so that the mesh M builds a boundary surface of a manifold. If the alpha value is set larger than B, boundary points are on the convex hull, so the mesh cannot be concave. If the alpha value is set smaller than A, some sample points are isolated from in the solid, so the reconstructed shape is not complete. Those both extreme cases are not interesting for tree crowns. Therefore, α must lay in interval $[A, B]$. Finding the proper α value is an iterative process. We initialize α as the average value of A and B. In each iteration step, we check if the boundary triangles constitute a manifold surface; if so, the alpha value can be reduced, if not, it is increased.

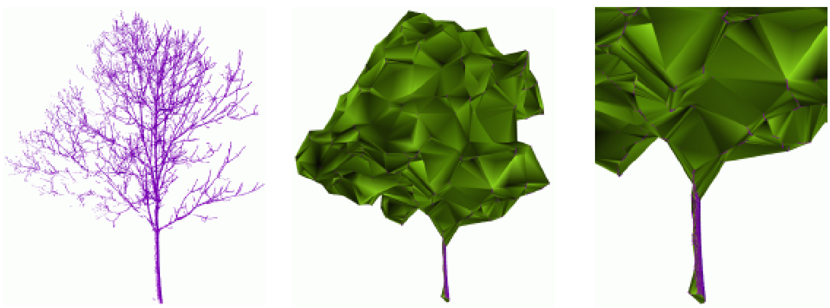
Figure 2 shows the pipeline of this approach.

5 Experiments and Discussion

Our algorithm is written with C Language with the support of OpenGL for graphics. Tests were held on a PC with P4, 3.0GHz processor and 1G RAM. CGAL library is used to perform Delaunay triangulation [24]. Our experimental results of concave tree crowns are shown with local illumination.

(a) (b) (c)

Fig. 3. Extraction of the Crown shape of a Maple tree; (a) is the source point cloud data; (b) is the boundary point cloud data; (c) is the extracted boundary mesh model



(a) (b) (c)

Fig. 4. Extraction of the crown shape of a Candlenut tree; (a) is the source point cloud data displayed with a cube for each point; (b) is a comparison of the extracted boundary mesh with the source point cloud date; (c) is a close view of (b)

We reconstruct the shape of tree crowns with two data sets of two trees. The first one is a single scan of a 20-meter high maple tree with leaves. Figure 3 (a) shows the original point model of the maple tree of 114997 points displayed with a cube for each point. When the alpha value is set as 4.2354, we acquire 2810 points on the boundary (Figure 3 (b)). Figure 3 (c) shows the reconstructed tree crown mesh model.

The second example is a candlenut tree without leaves shown in Figure 4. The original data of the candlenut tree has 86675 points (Figure 4 (a), and when the alpha value is set as 0. 41399, 4291 points are left on the boundary. The implementation of our algorithm is shown in Table 1, where the last column is the time spent from data input, to Delaunay triangulation, and to the list of all triangular faces on the boundary.

To show the properness of this approach, the original point model of the candlenut tree is combined to its reconstructed crown mesh model. Figure 4 (b) shows this comparison, and Figure 4 (c) shows a close view of Figure 4 (b). It can be seen in

Table 1. Experimental details on two data sets

Tree	Point set	Alpha value	Points on boundary	Time in secs
Maple	114997	4.2354	2810	1814.16
Candlenut	86675	0.41399	4291	2131.03

Figure 4 (b) and Figure 4 (c) that the reconstructed crown mesh model includes the original point model well.

These two examples show that the shape concavity is well reconstructed. The approach is illustrated here on both dense crown and spare (unfoliaged) one.

6 Conclusion

Current 3D acquisition systems lead to model more and more 3D shapes of real life objects. However, nowadays reconstruction approaches classically fail on high complexity objects, such as trees. Even if nice progresses have been noticed on the main branch structure on un-foliaged trees, the overall reconstruction is not satisfactory, especially on small structures and leaves.

We proposed hereby a method to reconstruct in 3D the scanned tree crown, in order to constrain the definition of the branch structures, especially the thinner ones, and contribute to define local geometrical constraints for leaf area reconstruction.

The principle of our approach is based on the use of the alpha-shape on the range point data set, a generalization of the convex hull and subgraph of the Delaunay triangulation. In the Delaunay triangulation process, we choose the triangle candidates on the boundary according to the alpha value, and constrain the surface mesh to stay a manifold. Therefore, our constructed boundary mesh builds in fact the silhouette of the crown. This shape of the tree crown is much more convincing than the convex hull of the tree crown in keeping the major concave features of the crown. This shape can be used to constrain faithfully the reconstruction of branches and foliage.

The proposed approach was successfully implemented and tested on two data sets.

Of course, the reconstructed crown shape mesh is rough, thus fast to render, and thus not strongly concave, so that higher branching structures are not recreated. In future, progress can be achieved by dividing the data into several subsets according to point density, with different alpha values applied to each subset. Concave silhouette surfaces can then be reconstructed independently, and then merged to a more detailed shape.

It is also interesting to note that such crown shapes do find applications in various domains. Such tree crown can contribute to define intermediate LOD plant models, from real plants or simulated ones. It contributes to define low weighted geometrical models. Of course, appropriate color and transparency value computations can increase the appearance while rendering such shapes.

Finally, the proposed technique may be of interest on a wide range of complex object, showing high topological complexity, where simplified representation, based on internal complex structure is useful. Such could be the case of human organs representation build from their internal vessels.

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