Region-based image segmentation with local signed difference energy

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Intensity inhomogeneity often causes considerable difficulties in image segmentation. To tackle this problem, we propose a new region-based level set method. The proposed method considers the local image information by describing it as a novel local signed difference (LSD) energy, which possesses both local separability and global consistency. The LSD energy term is integrated into an objective energy functional, which is minimized via a level set evolution process. Extensive experiments are performed to evaluate the proposed method, showing improvements in both accuracy and efficiency, as compared with the state-of-the-art approaches.

1. Introduction

Image segmentation involves a process that simplifies an image by partitioning the image domain into several regions. The segmentation accuracy and efficiency are crucial in many subsequent applications, such as modeling, visualization and quantification. However, the performance of segmentation is greatly influenced by a variety of potential difficulties. Among them, one major difficulty is intensity inhomogeneity. The key issue addressed in this paper is to segment the images with intensity inhomogeneities.

1.1. Previous work

Level set method has been extensively applied to image segmentation because it holds several desirable advantages over traditional methods, such as, sub-pixel accuracy and topology variability. Generally speaking, existing level set based segmentation methods can be classified into two groups, i.e., the edge-based methods (Kass et al., 1988; Osher and Sethian, 1988; Xu and Prince, 1998; Caselles et al., 1995; Yezzi et al., 1997; Caselles et al., 1997; Paragios and Deriche, 2000; Malladi et al., 1995; Paragios and Deriche, 2002) and the region-based methods (Chan and Vese, 2001; Tsai et al., 2001; Vese and Chan, 2002; Nikolova et al., 2006; Unger et al., 2008; Goldstein et al., 2010; Zhang et al., 2010) and the local methods (An et al., 2007; Li et al., 2008; Wang et al., 2009; Zhang et al., 2010). One of the most popular global methods is the piecewise constant model (or called the CV method) proposed in (Chan and Vese, 2001), which is derived from the Mumford–Shah segmentation technique (Mumford and Shah, 1989). The principle assumption under this method is that image intensities are statistically homogeneous in each segmented region. As reported in (Chan and Vese, 2001), this method can automatically detect all contours. However, the CV method has following two major problems.

The first problem is that the CV method is not convex. Hence, any meaningful solution to the CV method will only be a local minima, which makes the segmentation result may be somewhat sensitive to the initialization. To tackle this difficulty, in (Nikolova et al., 2006), Nikolova et al. proposed a Rudin–Osher–Fatemi (ROF) total variation based CV method (or named the TVCV method). Some extensions are proposed in (Unger et al., 2008; Goldstein et al., 2010). In (Goldstein et al., 2010), Goldstein et al. proposed the split Bregman method to solve the TVCV method.
The second problem is that the CV method often gives inaccurate segmentation results, especially when the images have great intensity inhomogeneities. Many local methods are proposed to solve this problem. The local methods assume that the intensities in a relatively small local region are separable, though these pixels may overlap globally. One of the effective local methods is proposed by Li et al. (2008), which introduced a local binary fitting (LBF) energy by a kernel function. Motivated by Li et al. (2008), the local Gaussian fitting energy and the local image fitting (LIF) energy have been proposed in (Wang et al., 2009 and Zhang et al., 2010), respectively. Improved performance had been obtained by these methods as compared with the global methods. However, the local methods are always sensitive to the initial contours, which makes them produce significant segmentation errors.

1.2. The method

In this work, we consider two properties, i.e., local separability and global consistency, to interpret the advantages and disadvantages of the global and local methods. In order to simultaneously hold the two properties, we propose a novel local signed difference (LSD) energy. We integrate the LSD energy into an objective energy functional, which is minimized via a level set evolution process. Specifically, the advantages of our method can be highlighted as follows.

1. The LSD energy adopts local image information to model the segmentation problem. As a result, compared with the global methods, our method handles intensity inhomogeneity better. Moreover, the LSD energy further considers the order of local clusters. Based on this, the consistencies of local clusters can be held well, which makes our method very stable. Hence, compared with the local methods, our method is less sensitive to the initialization. Experimental results also indicate our method does not produce serious segmentation errors.

2. The data force term derived from the LSD energy is computed once. However, in other local methods, the data force terms need to be recalculated in each iteration. Hence, the computational cost of our method is lower than other local methods. Experimental results have shown that the speed of our method is as fast as the CV method (Chan and Vese, 2001), which is faster than the local methods.

The rest of this paper is organized as follows. A brief summary of the global and local methods is presented in Section 2. Section 3 introduces two properties, i.e., local separability and global consistency. Section 4 describes the proposed segmentation method in detail. Section 5 gives some experiments and results. Discussions and conclusions are given in Section 6.

2. The global and local segmentation methods

Let \( \Omega \subset \mathbb{R}^2 \) be the two-dimensional image domain, and \( I: \Omega \rightarrow \mathbb{R} \) be the given gray-level image. Denote \( I_x = I(x) \), where \( x \in \Omega \), is the pixel value at the location of \( x \). The CV method proposed in (Chan and Vese, 2001) formulates image segmentation as a problem of finding a contour \( C \), which segments the image into two non-overlapping regions, i.e., the region inside the contour as \( \Omega_f \) (foreground) and the region outside the contour as \( \Omega_b \) (background). By specifying two constants \( \epsilon^f \) and \( \epsilon^b \), the global fitting energy is defined as

\[
E_g(C) = \lambda_f \int_{\Omega_f} |I_x - \epsilon^f|^2 \, dx + \lambda_b \int_{\Omega_b} |I_x - \epsilon^b|^2 \, dx,
\]

where \( \lambda_f, \lambda_b \) are the positive constants. As expressed in Eq. (1), the constants \( \epsilon^f \) and \( \epsilon^b \) globally control all the pixels in \( \Omega_f \) and \( \Omega_b \), respectively. They can be regarded as the global clusters, i.e., the foreground cluster and the background cluster. From this view, the CV method is a two-class clustering method.

Motivated by Chan and Vese (2001) and Li et al. (2008) proposed a local binary fitting (LBF) energy to separate the local foreground and background pixels, given by

\[
E_{bf}(C, C^f, C^b) = \lambda_f \int_{\Omega_f} K_{\sigma_f}(y) |I(y) - C^f|^2 \, dy + \lambda_b \int_{\Omega_b} K_{\sigma_b}(y) |I(y) - C^b|^2 \, dy,
\]

where \( K_{\sigma_f}(y) = \frac{1}{2\sqrt{\pi \sigma^2}} \exp \left[ -\frac{y^2}{2\sigma^2} \right] \) is the Gaussian kernel function parameterized with the scale parameter \( \sigma \). The total energy is defined as the integration on the whole image domain, given by

\[
E_l(C, C^f, C^b) = \int_{\Omega} E_{bf}(C, C^f, C^b) \, dx.
\]

Using the steepest descent method proposed in (Aubert et al., 2006) to minimize Eq. (3), we can obtain the local clusters \( C^f \) and \( C^b \) as follows:

\[
C^f = \frac{\int_{\Omega_f} K_{\sigma_f}(y) I(y) \, dy}{\int_{\Omega_f} K_{\sigma_f}(y) \, dy}, \quad C^b = \frac{\int_{\Omega_b} K_{\sigma_b}(y) I(y) \, dy}{\int_{\Omega_b} K_{\sigma_b}(y) \, dy}.
\]

Similarly, \( C^f \) and \( C^b \) can be interpreted as the local foreground cluster and the local background cluster at the location \( x \).

3. Local separability and global consistency

In this paper, we express the main differences between the global methods and the local methods as the differences of modeling local separability and holding global consistency.

For the local methods, if we restrict the foreground clusters and the background clusters into the constants, that is, \( C^f = \epsilon^f | x \in \Omega \) and \( C^b = \epsilon^b | x \in \Omega \), the local methods degenerate into the global ones. From this view, the global methods can be considered as a special case of the local methods. The only difference is that the global methods utilize a more strict global consistency constraint, which requires all the foreground clusters to be the constant (the same for the background clusters). Therefore, compared with the local methods, the global ones hold global consistency better. In practice, the global methods are more stable, or less sensitive to initialization.

On the other hand, the local methods consider the segmentation problem locally, and combine all the local information together. However, the global methods only use two global clusters to describe the segmentation problem. Hence, compared with the global methods, the local ones can model local separability better. Practically, the image details, such as weak boundaries, can be segmented better by the local methods.

To summarize, local separability and global consistency have their own contributions on the image segmentation. Local separability helps to tackle the intensity inhomogeneous problem, while global consistency makes the segmentation result insensitive to the initialization.

4. The method

4.1. The local signed difference (LSD) energy

As interpreted in Section 3, the global and local methods have their own advantages and disadvantages in modeling local separability and holding global consistency. To achieve better segmentation, it is reasonable to combine the two types of methods together.

In this paper, we propose a new local signed difference (LSD) energy, which not only models local separability well, but also...
takes global consistency into consideration. The LSD energy is described by the signed difference between the local background cluster $C_b^k$ and the local foreground cluster $C_f^k$, given by

$$ E_{ld}(x) = \text{sgn} \left( C_b^k - C_f^k \right) \frac{C_b^k - C_f^k}{|C_b^k|^k}. \quad (5) $$

where $\text{sgn}(x) = \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases}$ From Eq. (5), we see that the LSD energy is composed of two terms, i.e., the sign term $\text{sgn}(C_b^k - C_f^k)$ and the difference term $|C_b^k|^k - C_f^k|$. On the one hand, the sign term $\text{sgn}(C_b^k - C_f^k)$ shows that the local clusters $C_b^k$ and $C_f^k$ have an order. As discussed in (Wang et al., 2010), the order can help holding global consistency. The LSD energy is defined as follows:

$$ E_{ld}(x) = C_b^k - C_f^k. \quad (6) $$

Combining Eq. (6) with Eq. (4), we can get that

$$ E_{ld}(x) = \int_{C_b} \frac{K_{\sigma x}(y)l(y)dy}{J_0 \int_{C_b} K_{\sigma x}(y)dy} - \int_{C_f} \frac{K_{\sigma x}(y)l(y)dy}{J_0 \int_{C_f} K_{\sigma x}(y)dy}. \quad (7) $$

In our method, the kernel function $K_{\sigma x}$ is redefined as

$$ K_{\sigma x}(y) = \begin{cases} \frac{1}{\sqrt{2\pi} \sigma^2} \exp \left( -\frac{r^2}{2 \sigma^2} \right), & |y - x| < 2\sigma \\ 0, & |y - x| > 2\sigma \end{cases}, \quad (8) $$

where $c_k$ is a normalization constant.

### 4.2. The total energy

The total local energy is defined as the weighted integration on the whole image domain $\Omega$, given by

$$ E_l(\mathcal{C}) = \int_{\Omega} W(x) E_{ld}(x)dx. \quad (9) $$

where $W(x)$ is the weight of the pixel at the location $x$. The weight $W(x)$ is used to control the evolution of the contour $\mathcal{C}$, so that its evolution is within a narrow band (Sethian, 1996). The narrow band is defined as the region around the evolution contour $\mathcal{C}$.

To ensure that the evolution of the contour $\mathcal{C}$ is within the narrow band, the weight $W(x)$ should satisfy following two conditions.

- **Condition 1**: if the pixel location $x$ is not in the narrow band, $W(x)$ should be zero.
- **Condition 2**: if the pixel location $x$ is in the narrow band, $W(x)$ should be positive.

In our work, the weight $W(x)$ is defined by

$$ W(x) = \int_{C_b} K_{\sigma x}(y)dy \int_{C_f} K_{\sigma x}(y)dy. \quad (10) $$

The weight $W(x)$ satisfies the above two conditions (see A). Combining Eq. (7), Eq. (9) and Eq. (10), the LSD energy can be rewrote as follows:

$$ E_l(\mathcal{C}) = \int_{\Omega} \left( \int_{C_b} K_{\sigma x}(y)l(y)dy \int_{C_f} K_{\sigma x}(y)dy \right) dx. \quad (11) $$

It is necessary to incorporate the prior knowledge on the contour $\mathcal{C}$. One simple yet efficient method is to use the contour length prior, given by

$$ R(\mathcal{C}) = |\mathcal{C}|, \quad (12) $$

where $|\mathcal{C}|$ is the length of the contour $\mathcal{C}$. The contour length prior has been utilized in many level set methods due to the following two reasons. First, it is very easy to use the contour length prior. Second, the computational cost of the contour length prior is very small. Combining with the contour length prior, we can get the total energy as follows:

$$ E(\mathcal{C}) = E_l(\mathcal{C}) + \mu R(\mathcal{C}) = E_l(\mathcal{C}) + \mu |\mathcal{C}|, \quad (13) $$

where $\mu$ is a positive weighting constant. To handle the topological change of the segmentation contour, we will convert the total energy functional $E(\mathcal{C})$ to a level set formulation, which is described in the next subsection.

### 4.3. Level set formulation and energy minimization

The level set formulation is performed to solve the total energy $E(\mathcal{C})$ of Eq. (13). In the level set method, a contour $\mathcal{C}$ is represented by the zero level set of a Lipschitz function $\phi : \Omega \rightarrow \mathbb{R}$, which is called a level set function. Let $H(\cdot)$ be the Heaviside function, which is proposed in (Chan and Vese, 2001), then the total energy is defined as follows,

$$ E(\phi) = \int_{\Omega} \left( \int_{\Omega} K_{\sigma x}(y)H(\phi(y))l(y)dy \int_{\Omega} K_{\sigma x}(y)dy \\ - \int_{\Omega} K_{\sigma x}(y)l(y)dy \int_{\Omega} K_{\sigma x}(y)H(\phi(y))dy \right) dx + \mu L(\phi) + vP(\phi). \quad (14) $$

where $\mu$ and $v$ are the two positive weighting constants. Note that, in Eq. (14), the term $\int_{C_b} K_{\sigma x}(y)H(\phi(y))l(y)dy \int_{C_f} K_{\sigma x}(y)H(\phi(y))dy$ is eliminated. $L(\phi)$ is the contour length term, which is defined as follows:

$$ L(\phi) = \int_{\Omega} |\nabla H(\phi(x))|dx. \quad (15) $$

$P(\phi)$ is the regularization term proposed in (Li et al., 2005), which serves to maintain the level set function as a sign distance function, given by

$$ P(\phi) = \int_{\Omega} \frac{1}{2} (|\nabla \phi(x)| - 1)^2 dx. \quad (16) $$

As proposed in (Chan and Vese, 2001), the Heaviside function $H(x)$ is approximated by a smooth function

$$ H_r(x) = \frac{1}{2} \left[ 1 + \frac{2}{\pi} \arctan \left( \frac{x}{r} \right) \right]. \quad (17) $$

The corresponding derivation of $H_r(x)$ is defined as

$$ \delta_r(x) = H'_r(x) = \frac{\epsilon}{\pi(r^2 + x^2)}. \quad (18) $$

The parameter $\epsilon$ is set to 1.0. The standard gradient descent method is used to minimize the energy function $E(\phi)$ with respect to $\phi$ by solving the gradient flow equation, as follows:

$$ \frac{\partial \phi}{\partial t} = -\delta_r(\phi)DF + \mu \delta_r(\phi) \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + v \left( \nabla^2 \phi - \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \right). \quad (19) $$
The parameters for the comparative methods are set as follows. CV Chan and Vese, 2001, TVCV (Goldstein et al., 2010) and SBGFR method are set as follows: the scale \(D\) is referred as the data force term, as it is derived from the LSD.

\[
\text{D} \text{F} = \int_{\Omega} \left( K_{\sigma x}(y) \partial_y \int_{\Omega} K_{\sigma x}(y) dy \right) dx
- \int_{\Omega} \left( K_{\sigma x}(y) \int_{\Omega} I(y) K_{\sigma x}(y) dy \right) dx.
\]

\text{DF} is referred as the data force term, as it is derived from the LSD. 

4.4. The algorithm and computational complexity analysis

All the partial derivatives in Eq. (19) are simply discretized as the central finite differences, and the temporal derivative is discretized as a forward difference. The kernel \(K_{\sigma x}\) is truncated as an \(m \times m\) mask, where \(m = 4\sigma + 1\). The main procedure of our method is summarized as follows:

**STEP1.** Initialize the level set function \(\phi^0\) by Eq. (21), given by

\[
\phi^0 = \phi(x, t = 0) = \begin{cases}
-\rho & x \in \Omega_1 - \partial \Omega_l \\
0 & x \in \Omega_l \\
\rho & x \in \Omega - \Omega_l
\end{cases}
\]

where \(\partial \Omega_l\) is the boundary of \(\Omega\).

**STEP2.** Evolve the level set function \(\phi\) according to Eq. (19). To obtain the numeric solution of Eq. (19), the current level set \(\phi^{n+1}\) is updated by the previous iteration result \(\phi^n\), given by

\[
\phi^{n+1} = \phi^n + \Delta t \frac{\partial \phi^n}{\partial t}
\]

where \(\Delta t\) is the time-step.

**STEP3.** Repeat **STEP2** until the level set function \(\phi\) is converged or the maximum iteration number is reached.

For all the local methods, the most time-consuming step is the updating of the level set function \(\phi\), such as the **STEP2** in our method. This step depends on the calculation of the term of \(\frac{\partial \phi}{\partial t}\) and the time-step. Therefore, a small time-step is used for the LIF. Since the Gaussian filtering method is equivalent to the LIF, our method can hold global consistency well. Therefore, in our method, we see that the detailed information cannot be segmented by the CV, TVCV and SBGFR methods. For example, part of the background and foreground are mixed together on the real blood vessel images.

In some regions, especially those near the weak edges, the local methods, i.e., both LBF and LIF, perform better than the global methods. However, the local methods produce significant segmentation errors on other regions.

The failure reasons of the local and global methods are from two aspects. First, based on two global clusters, the global methods do not provide effective energies to model local separability. As the input images are inhomogeneous, they cannot segment out the weak boundaries accurately. Second, the local methods, including the LBF and LIF, do not use additional constraints to ensure that the segmentation result is globally consistent. Hence, their segmentation results contain some undesired boundaries. Take the second synthetic image (see Fig. 1(b)) as an example, parts of the segmentation contours are clearly away from the true boundaries (see the dashed yellow ellipses in Fig. 1(b)). To further interpret global consistency, we illustrate the images of the local clusters, i.e., \(C_l\) and \(C_b\), as shown in Fig. 2. From the input image, we see that the desired object is darker than its background. Hence, the output local foreground cluster \(C_l\) should be smaller than the local background cluster \(C_b\) on the entire image domain \(\Omega\), that is, \((C_l < C_b \forall x \in \Omega)\). However, in the classical local methods, parts of the local foreground clusters are larger than those of the local background clusters (see the blue dashed ellipses in Fig. 2). In our method, the LSD energy restricts the local clusters being correctly ordered. Based on this, our method can hold global consistency well. Therefore, in our method, the local foreground clusters are smaller (or darker) than the local background clusters.

5. Experimental results

A number of synthetic and real images are used to evaluate the performance of our method. The input images are normalized to \([0, 1]\). Unless otherwise specified, the parameters of the proposed method are set as follows: the scale \(\sigma = 3\), two weighting constants \(\mu = 0.002 \times 255 \times 255\), \(v = 1\), and the time-step \(\Delta t = 0.1\).

We compare our method with three global methods, namely, CV Chan and Vese, 2001, TVCV (Goldstein et al., 2010), and SBGFR (Zhang et al., 2010), and two local methods, namely, LBF (Li et al., 2008) and LIF (Zhang et al., 2010), quantitatively and qualitatively. The parameters for the comparative methods are set as follows:

1. For the CV method, the weighting constant \(\mu\) and the time-step \(\Delta t\) are set as the same as ours.
2. For the TVCV method, the contour length value is set according to the input image.
3. For the SBGFR method, the time-step \(\Delta t\) is set to 1. The SBGFR method needs not to specify \(\mu\) and \(v\).
4. For the LBF method, the scale \(\sigma\), the weighting constant \(\mu\), the weighting constant \(v\), and the time-step \(\Delta t\) are set as the same as ours.
5. For the LIF method, the scale \(\sigma\) and the time-step \(\Delta t\) are set to 3 and 0.025. As pointed out by this paper, the scale of the Gaussian kernel depends on the time-step. Therefore, a small time-step is used for the LIF. Since the Gaussian filtering method is used to regularize the level set function, the LIF method needs not to specify \(\mu\) and \(v\).

5.1. Visual comparisons with the state-of-the-art approaches

Fig. 1 presents the visual comparisons on two synthetic and two real blood vessel images. We need not to specify the initial contours for the TVCV method. As shown in this figure, although the input images are inhomogeneous and some boundaries are weak, our method can still segment the objects successfully. However, the detailed information cannot be segmented by the CV, TVCV and SBGFR methods. For example, part of the background and foreground are mixed together on the real blood vessel images.

In some regions, especially those near the weak edges, the local methods, i.e., both LBF and LIF, perform better than the global methods. However, the local methods produce significant segmentation errors on other regions.

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5.2. Comparisons of computational cost

The computational costs on the four images in Fig. 1 are also measured. We use a standard PC with a 2.4 GHz processor and 1024 MB of memory, and choose the MATLAB in all experiments. The computational cost is proportional to the iteration number. We use convergence of level set or reaching the maximum iteration as two termination criteria. In our experiments, we set the maximum iteration number to a large value, i.e., 1000, to ensure that the level set function is convergent before reaching the maximum iteration. Table 1 summarizes the comparison result. From this table, we can see that the computational cost of our method is lower.
than the classical local methods, especially on two real blood vessel images. In fact, our method is as fast as the TV method. The speeds of the TVCV and SBGFR are very fast. However, their segmentation results are not very good.

5.3. Comparisons of initialization sensitivity

We also compare our method with the state-of-the-art approaches on the sensitivity to the initialization. The tested images are the same as Fig. 1. For each input image, we perform four segmentation methods, i.e., the CV, LBF, LIF and our method, 100 times with different random initializations. To compare the segmentation accuracy, we utilize the $F$–score value as the evaluation criterion, given by

$$F - \text{score} = \frac{2 \cdot TP}{2 \cdot TP + FN + FP},$$

where $TP$, $FP$, and $FN$ are the true positive, false positive, and false negative, respectively. The ground truth is created manually.

![Fig. 1. Visual comparisons of the proposed method with the CV, TVCV, SBGFR, LBF, and LIF methods on two synthetic images ((a) and (b)) and two real blood vessel images ((c) and (d)). The initial and final contours are in green and in red, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image1)

![Fig. 2. Comparisons of local clusters.](image2)

Table 1: Comparisons of the computational costs (second).

<table>
<thead>
<tr>
<th></th>
<th>CV</th>
<th>TVCV</th>
<th>SBGFR</th>
<th>LBF</th>
<th>LIF</th>
<th>Our</th>
</tr>
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<tr>
<td>Fig. 1(a)</td>
<td>1.42</td>
<td>0.005</td>
<td>0.15</td>
<td>3.73</td>
<td>8.72</td>
<td>1.39</td>
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<tr>
<td>Fig. 1(b)</td>
<td>2.21</td>
<td>0.009</td>
<td>0.49</td>
<td>5.78</td>
<td>18.3</td>
<td>2.62</td>
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<td>Fig. 1(c)</td>
<td>2.35</td>
<td>0.011</td>
<td>0.35</td>
<td>31.7</td>
<td>9.81</td>
<td>2.97</td>
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<tr>
<td>Fig. 1(d)</td>
<td>2.52</td>
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<td>0.56</td>
<td>25.0</td>
<td>6.37</td>
<td>2.48</td>
</tr>
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</table>
Table 2 illustrates the comparisons in detail. Note that, each item in this table is composed of two terms, i.e., the mean and the variance. For example, the left-top item 0.773±0.000 indicates that the mean \( F/C_0 \) score value of the CV method in Fig. 1(a) image is 0.773, while the variance is 0.000. As shown in Table 2, the mean \( F/C_0 \) score values of our method are all larger than other three methods, showing that our method has higher segmentation accuracy. On the other hand, the variances of our method are lower than the local methods, which indicates that our method is less sensitive to the initialization.

The quantitative comparisons by the \( F/C_0 \) score values do not provide any visual insight on the segmentation results with the different initializations. In Fig. 3, we provide a visual example to evaluate how much these methods are sensitive to the initialization. As shown in this figure, the LBF and LIF methods produce significant segmentation errors, and their segmentation results are very different under different initializations. On the other hand, the results of the CV and our methods are almost independent of the initializations. However, the main limitation of the CV is that it cannot segment out the details, such as the weak boundaries (see the dashed blue ellipse). On the contrary, our method can segment out these details.

Both the CV method and our method hold global consistency well, so that the foreground and background clusters on the whole image can keep the same property. For example, in our method, the local foreground clusters and local background clusters have the same order. The CV method keep global consistency even better, since it only utilizes two global clusters. These global consistent properties can help enhancing the segmentation stability. Hence, although the contour evolution is local, stable segmentation results are achieved by both methods. However, as a global method, the CV method cannot model local separability well. Hence, the CV method cannot segment out the image details.

5.4. Comparisons of noise sensitivity

In this subsection, we evaluate the noise sensitivity of our method by comparing it with the state-of-the-art approaches. The noise image \( I_{\text{out}} \) is simulated by adding the Gaussian noise on the input image \( I_{\text{in}} \). That is, for each pixel, we have

\[
I_{\text{out}}(x) = I_{\text{in}}(x) + \beta n, \quad x \in \Omega
\]

(24)

where \( \beta \) is the noise level and \( n \) is a Gaussian variable following the standard Gaussian distribution.
We compare our method with the state-of-the-art approaches on 200 synthetic letter images. For each image, we simulate the five noise levels, namely, $\beta = \{0.01, 0.05, 0.1, 0.2, 0.3\}$. The numeric comparisons of the mean $F$-score values are shown in Table 3. As illustrated in this table, our results are better than those of other methods under the different noise levels. Fig. 4 presents two visual comparisons. From this figure, we see that our results are accurate and stable under the different noise levels.

5.5. Analysis of the parameters

It is necessary to examine the influence of the scale parameter $\sigma$ on the segmentation results of the proposed method. We apply our method to the same MR image with five different scale parameters. As shown in Fig. 5, all the segmentation results are very similar, meaning that our method is stable to the choice of the scale parameter $\sigma$. In general, with the small scales, the detailed boundaries are better segmented but the weak edges are ignored. The larger scale parameters result in the smoother segmentation contours.

6. Discussions and conclusions

6.1. Discussions

The capability of our LSD region-based segmentation method depends on following two aspects: (1) the object and the background should be piecewise smooth; (2) the object and the background should be distinguishable. In the following, we list some limitations of our method.

The first limitation is that it is hard to segment the wispy target. To decrease the effect of the noise, the scale parameter of our method cannot be set very small. In such case, small scale objects may fail to be segmented out. On the other hand, the main difficulty of the wispy target segmentation is to segment small scale objects.

The second limitation is that our method cannot segment the bright and dark targets simultaneously. The LSD energy incorporates the sign, which makes our method can only be applied to

<table>
<thead>
<tr>
<th>Noise level</th>
<th>CV</th>
<th>TVCV</th>
<th>SBGFR</th>
<th>LBF</th>
<th>LIF</th>
<th>Our</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.350</td>
<td>0.365</td>
<td>0.409</td>
<td>0.373</td>
<td>0.375</td>
<td>0.911</td>
</tr>
<tr>
<td>0.05</td>
<td>0.346</td>
<td>0.361</td>
<td>0.405</td>
<td>0.380</td>
<td>0.362</td>
<td>0.909</td>
</tr>
<tr>
<td>0.1</td>
<td>0.340</td>
<td>0.357</td>
<td>0.396</td>
<td>0.383</td>
<td>0.348</td>
<td>0.901</td>
</tr>
<tr>
<td>0.2</td>
<td>0.335</td>
<td>0.353</td>
<td>0.386</td>
<td>0.385</td>
<td>0.336</td>
<td>0.877</td>
</tr>
<tr>
<td>0.3</td>
<td>0.326</td>
<td>0.345</td>
<td>0.377</td>
<td>0.376</td>
<td>0.331</td>
<td>0.826</td>
</tr>
</tbody>
</table>

Fig. 4. Segmentation results under the five noise levels, i.e., $\beta = \{0.01, 0.05, 0.1, 0.2, 0.3\}$. We compare our method with the CV, TVCV, SBGFR, LBF, and LIF methods.
the cases where the object is either brighter or darker than the background. To overcome this difficulty, we can improve our method by using the multi-phase technique proposed in (Vese and Chan, 2002).

6.2. Conclusions

In this paper, we propose a novel local region-based level set segmentation method. The main contribution is that we propose the LSD energy, which not only models local separability, but also holds global consistency. Thereby, our method inherits the advantages of both global and local methods, e.g., the ability of handling image inhomogeneity, the robustness to initial contours and the low computational cost. The proposed method has been evaluated on a number of the images with intensity inhomogeneity, the robustness to initial contours and the low computational cost. The proposed method has been evaluated on a number of the images with intensity inhomogeneity, the robustness to initial contours and the low computational cost. The proposed method has been evaluated on a number of the images with intensity inhomogeneity, the robustness to initial contours and the low computational cost. The proposed method has been evaluated on a number of the images with intensity inhomogeneity, the robustness to initial contours and the low computational cost. The proposed method has been evaluated on a number of the images with intensity inhomogeneity, the robustness to initial contours and the low computational cost. The proposed method has been evaluated on a number of the images with intensity inhomogeneity, the robustness to initial contours and the low computational cost. The proposed method has been evaluated on a number of the images with intensity inhomogeneity, the robustness to initial contours and the low computational cost. The proposed method has been evaluated on a number of the images with intensity inhomogeneity, the robustness to initial contours and the low computational cost.

Acknowledgement

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Appendix A

Lemma Appendix A.1.

Let \( z = \int_{\Delta_j} K_{\sigma}(y) dy \), we get \( W(x) = z(1-z), (0 \leq z \leq 1) \). On the one hand, if the pixel location \( x \) is outside the narrow band, we have two cases: (1) \( z = 0 \) when \( x \) is outside the outer contour; and (2) \( 1 - z = 0 \) when \( x \) is inside the inner contour. In both cases, we get \( W(x) = 0 \). Thus, Eq. (10) meets Condition 1. On the other hand, if the pixel is within the narrow band, the contour \( c \) will split its local region into two non-empty sub-regions. Thus, we get \( 0 < z < 1 \), yielding \( W(x) > 0 \). Thereby, Eq. (10) meets Condition 2.

References


