PERSONALIZED VIDEO RECOMMENDATION BASED ON CROSS-PLATFORM USER MODELING

Zhengyu Deng[1,2], *Jitao Sang*[1,2], *Changsheng Xu*[1,2]

[1]National Lab of Pattern Recognition, Institute of Automation, CAS, Beijing 100190, China
[2]China-Singapore Institute of Digital Media, 139951, Singapore
{zydeng, jtsang, csxu}@nlpr.ia.ac.cn

ABSTRACT

Online propagation of videos has surged up to an unparalleled level. Most personalized video recommendation methods are based on single-platform user modeling, which suffer from data sparsity and cold-start issues. In this paper, we introduce cross-platform user modeling as a solution by smartly aggregating user information from different platforms. Unlike traditional recommendation methods where sufficient user information is assumed available in the target platform, this proposed method works well when there is little knowledge about users' interests in the target platform. While considering the difference of user behaviors in different platforms, on one hand, we enrich user profile in the target platform with related information in the auxiliary platform. On the other hand, we transfer the collaborative relationship defined in behaviors from the auxiliary platform to the target platform. Carefully designed experiments have demonstrated the effectiveness of the proposed method.

***Index Terms*—** Personalized video recommendation, cross-platform user modeling

1. INTRODUCTION

With the arising of Web2.0, online propagation of User Generated Content (UGC) has surged up to an unparalleled level, leading to the arrival of big data age. For example, the most popular online video sharing website YouTube [1], hosts almost 2 billion videos and in every minute, there are still more than 60 hours of new videos being uploaded to the site [1]. The tremendous data makes the exploration and discovery of new or interesting sources a daunting task. Therefore, personalized service (e.g. search, subscription and recommendation), plays a more and more important role in tackling the problem of information overload.

Personalized service is based on user modeling, which requires abundant data to understand user interest exactly. Currently, most of the user modeling strategies are based on single platform [2][3][4]. However, many restrictions are imposed on obtaining user private data and the available user information in one single platform is limited, which deteriorates the notorious "cold-start" problem. On the other hand, many network users create and maintain multiple accounts across different web2.0 platforms. User's behaviors on different platforms reflect the user's preference from different prospectives and jointly contribute to in-depth user understanding. Therefore, cross-platform user modeling by aggregating user information from different platforms will address the cold-start challenge in single platform and result in improved personalized services [5]. For example, for an inactive user about whom we knows little in YouTube, if we recognized his/her explicit interest in Beckham from claimed profile or activities in other platforms, we can confidently recommend Beckham's new game videos to him/her in YouTube.

One obstacle in cross-platform user modeling is the acquisition of associated user accounts, i.e., which account in one platform and which in another platform correspond to the same user. Fortunately, many users are willing to provide their separate accounts in different platforms, when registering into social network sites or using social media account management tools (e.g. FriendFeed [2]). For example, many Google+ users share the URL links of their accounts on YouTube, Flickr, Twitter, etc. at their homepages. The information aggregation is a trend with the further development of social media, which makes the data of user account association easily available. This trend enables cross-platform user modeling and provides opportunities to advanced applications.

In this paper, we address the personalized video recommendation problem by introducing cross-platform user modeling. We use YouTube as the target platform where to perform the recommendation task, and Google+ [3] as the auxiliary platform where user information is transferred. Two strategies are designed to strengthen the understanding of user interest in the target platform: one is profile enrichment and the other is collaborative relationship transfer. In brief, profile enrichment is to directly enrich user profile using their related

[1] http://www.youtube.com/.

[2] http://friendfeed.com/.

[3] http://plus.google.com/.

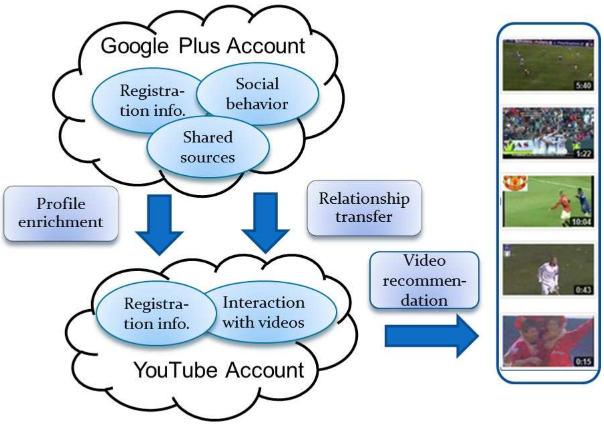

Fig. 1. The framework of our proposed approach.

profile information in the auxiliary platform. Collaborative relationship transfer is to transfer the behavior similarity of users from the auxiliary platform to the target platform. The overall user modeling is based on aggregation of the enriched profile and transferred relationship. The framework is illustrated in Fig. 1. The inputs include user profiles in Google+ and YouTube, and the output is the generated video recommendation list. According to the aforementioned two strategies, the framework contains three components, namely the social relationship transfer, the user profile enrichment and the video recommendation.

To summarize, the main contributions of this paper are as follows.

(1) We propose to tackle the personalized video recommendation problem by cross-platform user modeling. Data sparsity and cold-start issues are well addressed.

(2) Two strategies regarding enriching profile and transferring relationship are designed. User information from the auxiliary platform is effectively transferred to the target platform.

(3) We conduct experiments on a crawled real-world cross-platform dataset, where promising results are obtained. The improvement is significant especially for inactive users in the target platform.

2. DATA COLLECTION

In our experiments the users who have accounts in both Google Plus and YouTube were studied. We started from Google+ profile of which about one fourth contains URL link of user's YouTube account. For these users, we can crawl their YouTube profiles and other information. To collect the data, we randomly selected a user who has more than 100 friends as the seed and extend the dataset by identifying users who had commented the activities shared or released by the seed user. The identified users are then set as the new seed users and the crawling process is iteratively conducted. In this way, the crawled users maintain weak relations to one another. From Aug 2012 to October 2012, 71,613 Google+ user profiles are collected, which contains 17,212 YouTube links. Removing the invalid links, we obtained 6,292 YouTube user profiles finally. For this user set, the registration and activity information in both platforms were downloaded to construct our experimental dataset.

Table 1. Registration info. of a typical user in both platforms

YouTube	Google+
Recent activity 2012-05-19 **Registration** 2008-04-27 **Country** American	**Tagline** Blogger at TechCrunch. Has too many phones. **Introduction** Blogger at TechCrunch. Bragging rights It's not polite to brag. **Occupation** Blogger **Employment** TechCrunch Blogger, 2011 - present ReadWriteWeb Blogger, 2008 - 2011 **Gender** Female

Table 2. User behaviors in YouTube and Google+

YouTube	Google+
Upload videos	Release multi-modal sources
Favorite videos	Reshare multi-modal sources
Comment on channels or videos	Comment on posts

The registration information of a typical user in both Google+ and YouTube is displayed in Table 1. We can see that the registration information in YouTube is very sparse, while it's very abundant in Google+. This is reasonable by realizing the differences between social network and multimedia application platforms; social network is an interaction platform where users communicate with each other actively and are keen to share what they like with their friends, where users would like to introduce themselves in detail; while multimedia application platform is content-centric and most users do not bother to enrich their personal profiles. This phenomenon further strengthens the necessity of cross-platform user modeling. We also summarize the typical behaviors in Google+ and YouTube in Table 2, which shows that users focus on videos in YouTube while in Google+ users are involved in much richer multi-modal sources (article, photo, video).

The overlap of user profiles across different platforms is particularly interesting in the context of cross-platform user modeling. The more differently a user behaves in different platforms, the smaller overlap of the user profiles will be led [6]. Fig. 2 shows the overlap of tag-based profiles of the individual user between Google+ and YouTube platforms. In fact, for less than 20% of the users, their profiles have an overlap of more than 20%. The overlap of user profiles is small. In other words, user profile in Google+ platform only reflects a small part of the user characteristics in YouTube platform. This observation indicates that it's not applicable to aggregate user

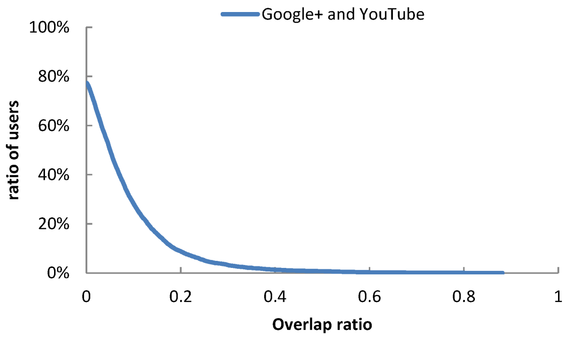

Fig. 2. Overlap of user profiles across platforms.

profiles in different platforms directly, which has to be done in an integrated way.

3. CROSS-SYSTEM USER MODELING

3.1. Social relation transfer

We assume that users who have similar profiles in Google+ are very likely to have similar profiles in YouTube, so we transfer the collaborative relationship in Google+ to YouTube. Furthermore, we want to find out the information elements in Google+ which reflect the characteristics of users in YouTube, so we model the user similarity in Google+ from different perspectives and give different weights to them. After that we pick up a dense subset of users in YouTube and take the similarity of users in the subset as supervision to obtain the weights.

Users on social network are associated with heterogeneous data. The challenge is how to effectively combine these data to model user similarity. Note that we can compute user similarity under different modalities [7], which is analogous to a kernel function in the kernel machines. This inspires us to adopt the multiple kernel learning (MKL) scheme [8] to integrate the multiple modalities, which is regarded as one of the principle way to combine heterogeneous data sources. Therefore, we adopt the state-of-the-art MKL algorithm to weight each modality [9].

We first discuss how to measure the similarity of Google+ users by defining a variety of kernel functions $\{K\}$ on different modalities of Google+ data. We then present a kernel learning technique to determine the optimal combination weights of multiple kernels by following the Kernel-Target Alignment (KTA) principle [10][9]. A series of candidate kernel functions on different modalities for measuring the similarity of Google+ users are presented as follows.

3.1.1. User similarity by registration information

Since registration information is abundant in Google+ and is adequate for user understanding, we take it as a modality to model user similarity. To represent the registration information of a user. We collect all the tags in registration information and build a tag space (dimension d). The tags of a user are converted into a feature vector by the traditional TF-IDF method. The user U_i can be represented by a vector $\mathbf{x_i} \in \mathbf{R^d}$. The normalized linear kernel to measure the user similarity is denoted as:

$$K^1(u_i, u_j) = \frac{\mathbf{x_i^T x_j}}{\sqrt{\mathbf{x_i^T x_i}}\sqrt{\mathbf{x_j^T x_j}}} \tag{1}$$

3.1.2. User similarity by comments

The interaction between a user and activities reflects the user preference. If two users have similar tagging behaviors, it's very likely that they have similar interests. Therefore, we also model user similarity by their comments on activities. We first collect all the comments for each user, and then represent them via the bag-of-word model. User similarity K^2 is modeled in the same way as Eq. 1.

3.1.3. User similarity by common activities

Google+ users often release and share videos, photos and articles. If two users share many common sources, they could be regarded to have similar interests. Meanwhile, we take the different modalities of sources into consideration and think that only certain sources are closely related to the user preference on videos; hence we model user similarity by videos, photos and articles separately. A simplified way is to count the number of the common sources as the kernel value. However, the gross of sources on Google+ is so tremendous that two users may have no common sharing even if they have similar interests. Therefore, we extract the tags associated with these sources and adopt the bag-of-word model to represent each user in specific domains, i.e., video, photo and article. And then the similarity $K^{3\sim5}$ is measured by cosine similarity as Eq. 1.

3.1.4. Optimal combination of multiple kernels

In the previous subsections, we define various kernel functions to measure user similarity in different modalities. Now we will find the optimal way to combine these modalities. As we want to obtain the user similarity in YouTube, we will give higher weights to the modalities that can reflect user characteristics in YouTube. In practice, linear combination is effective and robust; hence we will determine a linear combination of multiple kernels to fuse all modalities to measure user similarity, parameterized by a weight vector $\varphi \in \mathbf{R}^{N_k}$:

$$K(u_i, u_j; \boldsymbol{\varphi}) = \sum_{a=1}^{N_k} \varphi_a K^a(u_i, u_j) \tag{2}$$

where K^a is the kernel defined under the ath view of the users, and N_k is the number of modalities.

One straightforward method is to manually set the weights of different modalities, which however highly relies on domain knowledge and cannot get the optimal combination. In this section, we present a kernel-based learning technique to find the optimal combination of multiple kernels by following the principle of KTA [10][9].

As our goal is to model user similarity in YouTube from Google+, we select a dense subset of users (they have sufficient interaction behaviors with videos) from YouTube with size (N_u) and take user similarity in the subset as known relationship. Specifically, given the target matrix $\mathbf{Y}$ ($\mathbf{Y} \in \mathbf{R}^{N_u \times N_u}$), we adopt the kernel alignment [10] to measure the quality of kernel K with respect to the target matrix Y. Note that the kernel matrices need be centered [9] before kernel alignment and the step is as follows.

$$[\mathbf{K}]_{ij} = K_{ij} - \frac{1}{N_u}\sum_{i=1}^{N_u} K_{ij} - \frac{1}{N_u}\sum_{j=1}^{N_u} K_{ij} + \frac{1}{N_u^2}\sum_{i,j=1}^{N_u} K_{ij} \quad (3)$$

Let $\mathbf{K}, \mathbf{Y} \in \mathbf{R}^{N_u \times N_u}$ be two kernel matrices. Then the alignment between $\mathbf{K}$ and $\mathbf{Y}$ is defined by

$$\rho(\mathbf{K}, \mathbf{Y}) = \frac{\mathbf{E}[tr\mathbf{KY}]}{\sqrt{\mathbf{E}[tr\mathbf{KK}]\mathbf{E}[tr\mathbf{YY}]}} \quad (4)$$

Given the target graph represented by matrix $\mathbf{Y}$, we maximize the alignment ρ over $\mathbf{K}$ to solve the kernel. The matrix $\mathbf{Y}$ is observed from the YouTube platform. The solution of φ^* of the optimization problem is given by [9]

$$\boldsymbol{\varphi}^* = arg\ min \boldsymbol{\varphi}^T \boldsymbol{M} \boldsymbol{\varphi} - 2\boldsymbol{\varphi}^T \mathbf{b} \quad (5)$$

where $\mathbf{b}$ is the vector $[tr\mathbf{K^1Y}, ..., \mathbf{trK^{N_k}Y}]^\mathbf{T}$ and $\mathbf{M}$ is the matrix $[\mathbf{M}]_\mathbf{kl} := \mathbf{tr}\ \mathbf{K^k K^l}$, for $k, l \in N_k$.

3.2. User profiling

3.2.1. User profile in YouTube

Generally speaking, the registration information of users is very useful to analyze their preferences. Besides, users' active actions (like "upload" , "favor" or "add to playList") on videos strongly indicate their attentions and preferences as well. Therefore, the users' profiles could be built up by extracting the tags and categories associated with those videos as well as the registration information. However, the tags annotated by web users contain plenty of noises such as meaningless words or typos. To tackle this issue, we utilize WordNet to filter out the noises and only keep noun tags which are the least noisy representations for users' interests in YouTube.

For the representation of visual feature, we adopt the Spatial Pyramid Matching (SPM) model [11]. Specifically, we first obtain the key frame for each video, then we extract the local descriptors of Scale-Invariant Feature Transform (SIFT) for each image [12] [13]. All these descriptors are quantized into d_x groups by a K-means clustering process. Given an image, we assign each of its SIFT descriptors to a nearest cluster. Then each image is converted into a fixed length of feature vector $\mathbf{x} \in \mathbf{R^{d_x}}$, where d_x is the size of visual vocabulary. The ith component of this vector counts the frequency of SIFT descriptor assigned to cluster i. For all key frames, we get the maximum values at each position and obtain the final visual profile of a user. For each user, the profile is represented as

$$u_i := \{\Theta_T^i, \Theta_V^i\} \quad (6)$$

where Θ_T^i is the text profile and Θ_V^i is the visual profile of the user.

3.2.2. User profile enrichment

As analyzed in Section 2, the tag-clouds in different platforms are not accordant. The rough aggregation of all user information across different platforms is not applicable. However, many users tend to present themselves and illustrate their backgrounds and hobbies when they register in a new platform. Registration information strongly indicates user preference and could be considered as area-irrelevant. Therefore, these information can be utilized to enrich user profile. As observed, users are more likely to introduce themselves in social network platforms than social media application platforms; hence it's applicable to take user registration information from Google+ to enrich their profiles in YouTube. Besides, there are abundant behaviors of user interacting with multimodal sources in Google+, but only the behaviors that users interact (share or reshare) with videos could directly reflect user preferences in YouTube. Thus we extract these behaviors of users from Google+ to enrich their profiles in YouTube.

4. EXPERIMENTS AND ANALYSIS

4.1. Experimental settings

As mentioned in Section 2, we obtained 6,292 users who have both Google+ and YouTube profiles. The videos uploaded, favored or added into playList from Sep 2012 to Oct 2012 by these users are taken as the test data. Specially, we think that it's nonsense to conduct experiments on users who have too small or too many test videos. Therefore we select the users whose test videos is between 8 to 1,000 and obtain 1,022 users finally. We aggregate the test videos of these users as test collection and the total number of the test videos is 15,169. In our experiment, for a target user, we get a score for each video by the similarity with the user, and then we rank these videos according to the scores and generate a personalized ranking list for the target user and examine whether the videos in user's test data are ranked at top positions. Each video is represented by its tags, category and description using TF measure. Additionally, we also utilize SIFT feature and adopt the BoW model to represent visual information of the videos. Given a video v, its profile is $v := \{\mathrm{T}, \mathrm{V}\}$, where T is the text profile and V is the visual profile.

As we focus on solving the cold-start and sparsity issues, the user profiles on YouTube are eliminated in our experiments. In order to evaluate the performance, we test the strategies as follows:

(1) Recommend only by YouTube Profile (S1)

Given a user u, for each video v, the score is

$$p(v \mid u) = \lambda_T p(\mathrm{T} \mid \Theta_T) + \lambda_V p(\mathrm{V} \mid \Theta_V) \quad (7)$$

(2) Recommend by Profile Enrichment (S2)

$$p(v \mid u) = \lambda_T (p(\mathrm{T} \mid \Theta_T) + p(\mathrm{T} \mid \Theta_{T'})) + \lambda_V p(\mathrm{V} \mid \Theta_V) \quad (8)$$

where $\Theta_{T'}$ is the user profile in Google+ platform.

(3) Recommend by YouTube Profile with Collaborative Transfer (S3)

$$\begin{aligned} p(v \mid u) =& \lambda_{cb}(\lambda_T p(\mathrm{T} \mid \Theta_T) + \lambda_V p(\mathrm{V} \mid \Theta_V)) \\ &+ (1-\lambda_{cb})(\lambda_T \frac{\sum_j K(u,u_j) p(\mathrm{T} \mid \Theta_T^j)}{\sum_j K(u,u_j)} \\ &+ \lambda_V \frac{\sum_j K(u,u_j) p(\mathrm{V} \mid \Theta_V^j)}{\sum_j K(u,u_j)}) \end{aligned} \quad (9)$$

where $K(u, u_j)$ is the similarity between u and u_j in Google+ platform.

(4) Recommend by Profile Enrichment with Collaborative Transfer (S4).

$$\begin{aligned} p(v \mid u) =& \lambda_{cb}(\lambda_T (p(\mathrm{T} \mid \Theta_T) + p(\mathrm{T} \mid \Theta_{T'})) + \lambda_V p(\mathrm{V} \mid \Theta_V)) \\ &+ (1-\lambda_{cb})(\lambda_T \frac{\sum_j K(u,u_j) p(\mathrm{T} \mid \Theta_T^j, \Theta_{T'}^j)}{\sum_j K(u,u_j)} \\ &+ \lambda_V \frac{\sum_j K(u,u_j) p(\mathrm{V} \mid \Theta_V^j)}{\sum_j K(u,u_j)}) \end{aligned} \quad (10)$$

In the previous formulations, $p(\mathrm{T} \mid \Theta_T)$ and $p(\mathrm{V} \mid \Theta_V)$ are computed by cosine similarity (see Eq. 1). The performance assessment measure is F-score.

In order to obtain the linear weights of kernels (see Eq. 5), we select 131 dense users in YouTube and adopt the bag-of-word model to represent each user. And then we employ the cosine similarity to measure the user similarity.

4.2. Experimental result and analysis

The learnt weights of the kernels are presented in Table 3. From this table we can see that the shared articles and the registration information are more useful to model user similarity in YouTube, while the shared photos and the comments are less effective.

Table 3. The linear parameters by KTA

Kernel	1	2	3	4	5
Name	Registration	Comment	Video	Photo	Article
Weight	0.266	0.125	0.204	0.020	0.385

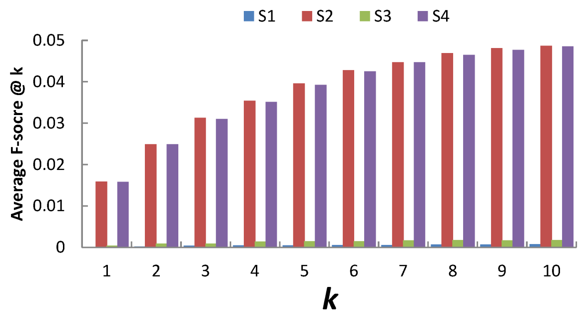

Fig. 3. The performance of different strategies.

In our experiments, we first tuned the values in [0,1] with interval of 0.1 and got the optimal range within [0.9,1]. Then we change the interval to 0.01 and tuned again in [0.9,1] to obtain the final optimal values: $\lambda_{cb} = 0.95$, $\lambda_T = 0.98$ and $\lambda_V = 0.02$. The comparison of average F-score at different depths by different strategies is illustrated in Fig. 3. It shows that the strategy that enrich user profile with part of Google+ information has the best performance, while collaborative transfer has little effect. In order to ascertain whether this phenomenon is due to the collaborative relationship transferred is from a different platform, we have designed another experiment. Firstly, we select a dense subset with 601 users from YouTube and conduct a experiment on these users. And then we use collaborative relationship in YouTube directly instead of utilizing user similarity in Google+. The result is displayed in Fig. 4, where S5 denotes the strategy 'Profile Enrichment with Collaborative relationship in YouTube', which also achieves inferior performance than profile enrichment (S2). Two conclusions are made from the results: 1) profile enrichment contributes much to the improvement of recommendation performance; 2) collaborative filtering is useful when user profile in the target platform is very sparse.

As analyzed in Section 2, user behaviors differ from platform to platform. Therefore, we assume that it's not feasible to directly aggregate all profiles of a user across platforms. We have designed a experiment to validate this hypothesis. We combine all the information in Google+ and YouTube for each user and obtain the compounded "Global Profile" for each of them. And then the video recommendation list is generated based on this profile. The performance is shown in Fig. 5. We can see that the performance by global profile is much worse than that by YouTube profile enriched with only certain information in Google+ (S2), which is consistent with

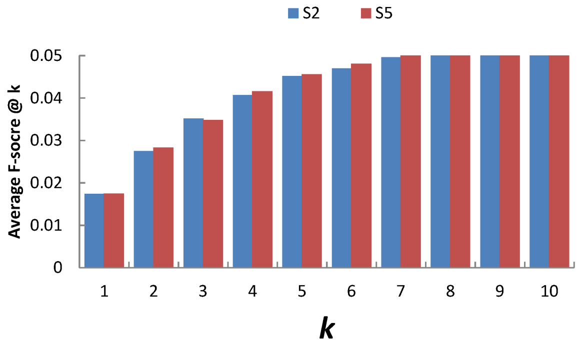

Fig. 4. The performance of utilizing user similarities in YouTube.

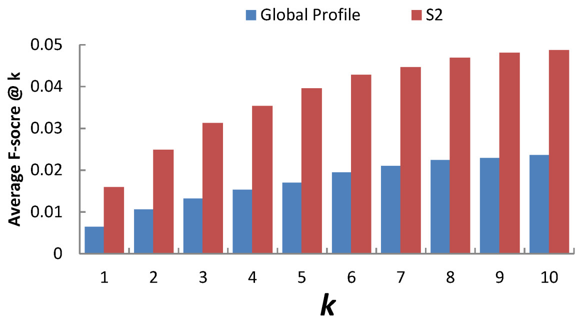

Fig. 5. The performance of profile enrichment.

our analysis and assumption.

5. CONCLUSIONS

In this paper, we have proposed a cross-platform user modeling method to address the cold-start and data sparsity issues in personalized video recommendation. By analyzing the difference of user behaviors across platforms, we presented two strategies: collaborative relationship transfer and profile enrichment. Promising experimental results have demonstrated that profile enrichment is more generally effective in improving recommendation performance, while collaborative relationship transfer serves as important complementation especially when the user original profile is extremely sparse. Our proposed method has been proved a valuable method to tackle the data sparsity and cold-start issues in the personalized recommendation problems. In the future we will be working towards enriching profile by deeper analysis of user behavior and applying the cross-platform solution framework to other personalized services.

6. ACKNOWLEDGEMENT

This work is supported in part by National Basic Research Program of China (No. 2012CB316304), National Natural Science Foundation of China (No. 61225009), and Beijing Natural Science Foundation (No. 4131004). This work is also supported by the Singapore National Research Foundation under its International Research Centre@Singapore Funding Initiative and administered by the IDM Programme Office.

7. REFERENCES

[1] Encyclopedia, “http://en.wikipedia.org/wiki/youtube/,” .

[2] F. Abel, S. Araújo, Q. Gao, and G.J. Houben, “Analyzing cross-system user modeling on the social web,” *Web Engineering*, pp. 28–43, 2011.

[3] Qi Gao, F. Abel, G. Houben, and Ke Tao, “Interweaving trend and user modeling for personalized news recommendation,” in *Web Intelligence and Intelligent Agent Technology (WI-IAT), 2011 IEEE/WIC/ACM International Conference on*, aug. 2011, vol. 1, pp. 100 –103.

[4] Z. Xu, Y. Zhang, Y. Wu, and Q. Yang, “Modeling user posting behavior on social media,” in *Proceedings of the 35th international ACM SIGIR conference on Research and development in information retrieval*. ACM, 2012, pp. 545–554.

[5] Ming Yan, Jitao Sang, Tao Mei, and Changsheng Xu, “Friendtransfer: Cold-start friend recommendation with cross-platform transfer learning of social knowledge,” in *International Conference on Multimedia and Expo (ICME)*. IEEE, 2013.

[6] F. Abel, Q. Gao, G.J. Houben, and K. Tao, “Analyzing user modeling on twitter for personalized news recommendations,” *User Modeling, Adaption and Personalization*, pp. 1–12, 2011.

[7] J. Zhuang, T. Mei, S.C.H. Hoi, X.S. Hua, and S. Li, “Modeling social strength in social media community via kernel-based learning,” in *Proceedings of the 19th ACM international conference on Multimedia*. ACM, 2011, pp. 113–122.

[8] G.R.G. Lanckriet, N. Cristianini, P. Bartlett, L.E. Ghaoui, and M.I. Jordan, “Learning the kernel matrix with semidefinite programming,” *The Journal of Machine Learning Research*, vol. 5, pp. 27–72, 2004.

[9] C. Cortes, M. Mohri, and A. Rostamizadeh, “Two-stage learning kernel algorithms,” in *Proceedings of the 27th International Conference on Machine Learning*, 2010, pp. 239–246.

[10] N. Shawe-Taylor and A. Kandola, “On kernel target alignment,” in *Advances in Neural Information Processing Systems 14: Proceedings of the 2001 Neural Information Processing Systems (NIPS) Conference*. The MIT Press, 2002, vol. 1, p. 367.

[11] Svetlana Lazebnik, Cordelia Schmid, and Jean Ponce, “Beyond bags of features: Spatial pyramid matching for recognizing natural scene categories,” in *CVPR (2)*, 2006, pp. 2169–2178.

[12] D.G. Lowe, “Distinctive image features from scale-invariant keypoints,” *International journal of computer vision*, vol. 60, no. 2, pp. 91–110, 2004.

[13] Chunjie Zhang, Jing Liu, Qi Tian, Changsheng Xu, Hanqing Lu, and Songde Ma, “Image classification by non-negative sparse coding, low-rank and sparse decomposition,” in *Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*. IEEE, 2011, pp. 1673–1680.