

Generalized Quotient Image

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Abstract

In this paper, we present a unified framework for modeling intrinsic properties of face images for recognition. It is based on the quotient image (QI) concept, in particular on the existing works of QI [1, 2], Spherical Harmonic [13, 14, 15], [16, 17], Image Ratio [3, 5, 6, 7] and Retinex [4, 9]. Under this framework, we generalize these previous works into two new algorithms: (1) Non-Point Light Quotient Image (NPL-QI) extends QI to deal with non-point light sources by modeling non-point light directions using spherical harmonic bases; (2) Self-Quotient Image (S-QI) extends QI to perform illumination subtraction without the need for alignment and no shadow assumption. Experimental results show that our algorithms can significantly improve the performance of face recognition under varying illumination conditions.

1 Introduction

Quotient image designed for dealing with illumination changes in face recognition by Shashua and Riklin-Raviv [1, 2], is a simple yet practical algorithm for extracting illumination invariant representation.

It has been shown that the quotient image, i.e. image ratio between a test image and linear combination of three images illuminated by non-coplanar lights, depends only on the albedo information, and therefore is illumination free. This method is practical for recognizing face under varying illumination conditions in the sense that it requires only one template image for each person, and that it does not need a training set for each person because it assumes the same 3D geometry for all persons. Several assumptions are made regarding the facial shape, absence of shadow, and knowledge about alignment between images for QI calculation.

Besides the QI method, many others techniques have been proposed for face representation under various illumination conditions in recent years.

Belhumeur etc. [10, 11, 12] prove that face images

with the same pose under different illumination conditions form a convex cone, called illumination cone. Ramamoorthi [13, 14, 15] and Basri etc. [16, 17] independently apply the spherical harmonic representation to explain the low dimensionality of differently illuminated face images. The synthesis and recognition results of illumination cone and spherical harmonics cast light on robust face recognition under various illuminations. However their application range is limited by needing 3D face model.

Similar to the image ratio technique in QI, Nayar and Rolle [5], and Jacobs et al [3] also introduce similar algorithms for face image intrinsic property extraction. However both of the two groups only analyze this approach by Lambertian model without shadow. Based on Land's Retinex [9], Jobson et al [6, 7] and Gross and Brajovic [8] develop reflectance estimation method by the ratio of original image and its smooth version. The difference of the two Retinex-based algorithms is that Jobson's filter is isotropic and Gross and Brajovic's filter is anisotropic.

The motivation of our work is extracting intrinsic, illumination invariant features from a face image based on the QI technique. Firstly a generalized QI framework is proposed, in which no assumption on the type of light source and the absence of shadows is made. Then from this framework we derive two methods, Non-Point Light Quotient Image (NPL-QI) and Self-Quotient Image (S-QI). The NPL-QI extends QI to non-point light sources with spherical harmonic bases. And S-QI extends QI to perform illumination subtraction (de-lighting) without the need for alignment and no-shadow assumption as QI does. Experimental results show that our algorithms can significantly improve the performance of face recognition under varying illumination conditions.

The paper is organized as follows. In section 2, we review the most related works about QI and Illumination modeling. In section 3, we describe the intrinsic factor of face image and propose our generalized QI framework. Two new methods, NPL-QI and S-QI, are advanced in this section. The experiment results are presented in section 4. Fi-

nally a conclusion is made in section 5.

2 Existing Works

Quotient image is based on the Lambertian Model.

$$I = \rho n^T \bullet s \quad (1)$$

where ρ is the albedo (surface texture) of face, n^T is the surface normal (3D shape) of face (assume the same for all faces in quotient image algorithm), \bullet is the dot product, and s is the point light source, which can vary arbitrarily. Let I_1, I_2, I_3 are three non-collinearly illuminated images of face a and their corresponding lighting sources are s_1, s_2, s_3 respectively. Therefore any point light source s_y can be taken as the linear combination of s_i with coefficient x_i , $s_y = \sum_{j=1}^3 x_j s_j$, where $i = 1, 2, 3$. The quotient image Q_y of face y illuminated by light source s_y against face a is defined by

$$\begin{aligned} Q_y(u, v) &= \frac{\rho_y(u, v)}{\rho_a(u, v)} \\ &= \frac{\rho_y(u, v) n^T(u, v) \bullet s_y}{\rho_a(u, v) n^T(u, v) \bullet s_y} \\ &= \frac{I_y(u, v)}{\rho_a(u, v) n^T(u, v) \bullet \sum_{j=1}^3 x_j s_j} \\ &= \frac{I_y(u, v)}{\sum_{j=1}^3 x_j I_j(u, v)} \end{aligned} \quad (2)$$

where u and v range over the image.

From the above equation 2, we can conclude that the quotient image defined as the ratio between a test image I_y and a linear combination of three non-collinearly illuminated images I_j with the coefficients x_j , which simulates the lighting direction of I_y , is illumination free and it depends only on albedo information of the two faces y and a .

The following assumptions are made in the quotient image framework: (a) the imaging process follows the Lambertian model without shadow and face is illuminated by point light source; (b) all the faces under consideration have the same shape, i.e. the same surface normal; (c) accurate alignment between faces is known; and (d) a training set of faces under at least three non-collinear illuminations is available as basis for estimation of illumination directions.

However, in face recognition system the above assumptions are often not satisfied at the same time. The light sources are generally not of point; 3D face shapes of different people are not the same in general; the shadow can exist; and accurate alignment is still an unsolved problem by now.

Nayar and Rolle [5] also advance one kind of QI. This kind of QI is the ratio of an image point with its neighboring

points. According to their analysis under the assumption of Lambertian model, they deduce that this QI is the ratio of reflectance coefficients, which is illumination free.

Let I is the image and \hat{I} is its smooth version, Nayar and Rolle deduce their QI by

$$Q = \frac{I - \hat{I}}{I + \hat{I}} = \frac{\rho - \hat{\rho}}{\rho + \hat{\rho}} \quad (3)$$

where ρ_1 and ρ_2 are the albedo of I and \hat{I} respectively.

Our Self Quotient Image (S-QI) algorithm has similar form as that of Nayar and Rolle's, but we analyze the invariant properties of the image ratio between an image and its smooth version under all cases, including shading region, shadow region and edge region. Furthermore we introduce a simple edge preserving filter for getting smooth version of the original image.

Jacobs etc [3] also introduce a similar kind of QI, which is the ratio of two images. They show that for point sources and objects with Lambertian reflectance, the ratio of two images from the same object is simpler than the ratio of images from different objects. Let $z = f(x, y)$ is object surface and $(s_x, s_y, s_z), (l_x, l_y, l_z)$, are two point light sources. The QI, i.e. the ratio of the same object with the two light sources is

$$\begin{aligned} I &= \rho \frac{-(s_x, s_y, s_z) \bullet (f_x, f_y, 1)}{\sqrt{f_x^2 + f_y^2 + 1}} \\ J &= \rho \frac{-(l_x, l_y, l_z) \bullet (f_x, f_y, 1)}{\sqrt{f_x^2 + f_y^2 + 1}} \\ Q &= \frac{s_x f_x + s_y f_y + s_z}{l_x f_x + l_y f_y + l_z} \end{aligned} \quad (4)$$

If the two light sources illuminate two objects with surface $z = f(x, y)$ and $z = g(x, y)$, and albedo ρ_1, ρ_2 respectively, then the corresponding QI becomes

$$\begin{aligned} I &= \rho_1 \frac{-(s_x, s_y, s_z) \bullet (f_x, f_y, 1)}{\sqrt{f_x^2 + f_y^2 + 1}} \\ J &= \rho_2 \frac{-(l_x, l_y, l_z) \bullet (g_x, g_y, 1)}{\sqrt{g_x^2 + g_y^2 + 1}} \\ Q &= \frac{s_x f_x + s_y f_y + s_z}{l_x g_x + l_y g_y + l_z} \left(\frac{\rho_1 \sqrt{f_x^2 + f_y^2 + 1}}{\rho_2 \sqrt{g_x^2 + g_y^2 + 1}} \right) \end{aligned} \quad (5)$$

From the equation 4 and 5, QI of images from the same object is far simpler than the QI of images from two different objects. They transfer this simplicity into simpler properties, the summation of gradient over the entire QI region as shown in equation 6.

$$\iint \min(I, J) \|\nabla(\frac{I}{J})\| dx dy \quad (6)$$

As in Nayar’s approach, Jacobs’s method only considers the Lambertian model without shadow and the object surface is smooth, i.e. the partial differential exists.

The Retinex theory motivated by Land [4, 9] deals with illumination effects on images. According to his experiments, human vision can discriminate color under different lighting conditions. Jobson, et al [6, 7] present a multi-scale version of Retinex method for high quality visual display of high dynamic range image on low dynamic range devices, such as printer and computer screen. This is closely related to the illumination issue. More recently Gross and Brajovie [8] present an anisotropic version of Retinex for illumination normalization. Both of the research groups propose an algorithm which estimates low frequency component of the input image as the light field and compensated illumination variations by subtracting it from the input image. This approach is based on that image can be represented by the product of reflectance R and illumination L , shown in equation 7.

$$I(x, y) = R(x, y)L(x, y) \quad (7)$$

Estimating the L and R only from the input image I is the well-known ill-posed problem. Therefore there are two assumptions: (1) the illumination L is smooth and (2) the reflectance R can be varied randomly. Based on these assumption, Jobson et al employ an isotropic filter and Gross and Brajovie apply an anisotropic filter to get the smooth version of image I for the estimation of the L .

3. Generalized Quotient Image

Each image contains some intrinsic information of the 3D scene it represents, although it is not sufficient to represent the original object(s) with a single image. In this section, we develop a new representation from an input image aiming to separate the intrinsic property from the extrinsic one.

3.1 Intrinsic and Extrinsic Factorization

The following is a typical Lambertian model, which can be factorized into two parts:

$$I(u, v) = \rho(u, v)n(u, v)^T \bullet s = F \bullet s \quad (8)$$

In the above, $F = \rho n^T$ depends on the albedo and surface normal and hence is the intrinsic representation of an object. It is F that represents the identity of a face. s is the illumination and is the extrinsic factor. Current appearance-based methods including PCA etc [21] learn a representation from images I and hence mix the intrinsic factor for the identity with the extrinsic factor. This is one of the main problems in highly accurate face recognition. We conclude that separating the two factors and removing the extrinsic factor is a key to achieving robust face recognition.

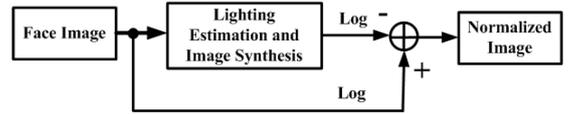


Figure 1: Generic Quotient Image Framework

3.2 Illumination Normalization

We generalize the existing QI methods described in Section 2 into a generalized framework, as shown in Figure 1. There are two main steps: (1) illumination estimation and (2) the illumination effect subtraction. First, the extrinsic factor is estimated and a synthesized image is generated. The synthesized face image has the same illumination and 3D shape as the input but different albedo. Then the illumination is normalized by taking the difference between the logarithms of the input and the synthesized images. Because the synthesized image has the same 3D shape and illumination with the original one, the normalized image is $(\log \rho_0 - \log \rho_1)$, where ρ_0 and ρ_1 are the albedo maps of the input and synthesized images, respectively; and is therefore illumination-free.

The logarithm is necessary because if the subtraction were between the two images directly, the result Q would be

$$Q = I - \hat{I} = (\rho_0 - \rho_1)n(u, v)^T \bullet s \quad (9)$$

which would still be illumination dependent. In the following, we derive two generalized QI algorithms, Non-Point Light Quotient Image (NPL-QI) and Self-Quotient Image (S-QI), from this framework.

3.3 Non-Point Light QI (NPL-QI)

Here, as in the original QI, we also assume that all the modeled object have the same 3D. NPL-QI takes the advantage of the linear relationship between spherical harmonic bases and PCA bases and extends the illumination estimation of QI from single point light source to any type of illumination conditions. Instead of explicitly estimating the 3D face shape, we replace the spherical harmonic bases with their linear transformed version, PCA bases. The quotient image of this method has the same invariant form, albedo ratio of two faces, as the original QI.

3.3.1 Analysis

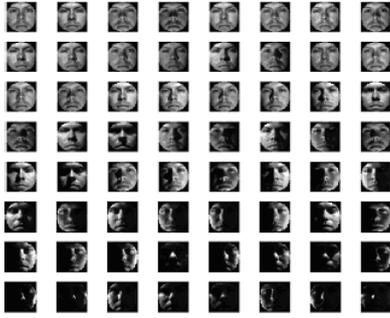
As we introduce in Section 1, the original QI method only works under the assumption of face image illuminated with single point light source without shadow. However the ordinary face images are always illuminated with non-point light sources and shadow will present unless the face is illuminated by frontal lighting.

According to Ramamoorthi [13, 14, 15] and Basri [16, 17]’s spherical harmonic representation, face image I can be represented by

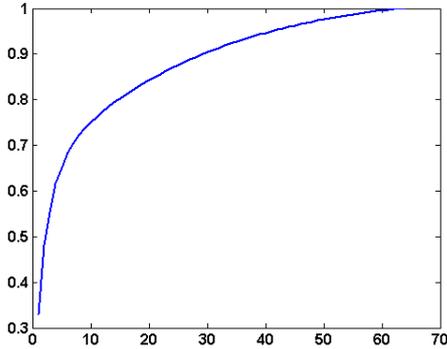
$$I = \rho H \bullet o \quad (10)$$

where $H = [h_1, h_2, \dots, h_n]$ is the spherical harmonic bases and o is the harmonic light. According to their finding, the first 9 harmonic bases, shown in equation 11, can well describe the image of diffuse object, such as human face.

$$\begin{aligned} h_1 &= \sqrt{\frac{1}{4\pi}} & h_2 &= \sqrt{\frac{3}{4\pi}}n_z \\ h_3 &= \sqrt{\frac{3}{4\pi}}n_x & h_4 &= \sqrt{\frac{3}{4\pi}}n_y \\ h_5 &= \frac{1}{2}\sqrt{\frac{5}{4\pi}}(n_z^2 - n_y^2 - n_x^2) & h_6 &= 3\sqrt{\frac{5}{12\pi}}n_zn_x \\ h_7 &= 3\sqrt{\frac{5}{12\pi}}n_zn_x & h_8 &= \frac{3}{2}\sqrt{\frac{5}{12\pi}}(n_x^2 - n_y^2) \\ h_9 &= 3\sqrt{\frac{5}{12\pi}}n_y n_x \end{aligned} \quad (11)$$



(a)



(b)

Figure 2: (a) face images (b) energy accumulative distribution for 64 dimensions

Because these bases must be calculated with known 3D geometry, the application range of this representation is limited. According to Ramamoorthi’s analysis [14], there is

linear relationship between PCA eigenvectors and spherical harmonic bases.

$$U = BT \quad (12)$$

where $U = [u_1, u_2, \dots, u_n]$ is the eigenvector matrix of a face under all lighting conditions and T is $n \times n$ transformation matrix. The equation 10 can be written as

$$I = B \bullet o = U \bullet l \quad (13)$$

where $l = T^{-1}o$.

If we get a densely sampled images, such as in Yale B (Figure 2(a)), we can get U , which is a well approximated linear-transferred spherical harmonic bases. Replacing these bases from PCA with three images in QI, we get NPL-QI

$$\begin{aligned} Q_y &= \frac{\rho_y}{\rho_a} \\ &= \frac{\rho_y \sum_i h_i o_i}{\rho_a \sum_i h_i o_i} \\ &= \frac{I_y}{U \bullet l} \end{aligned} \quad (14)$$

Because in some extremely illuminated face images there are obvious cast shadow, the first 9 eigenvectors can not carried over 95 % image energy, shown in Figure 2. Therefore more eigenvectors are needed in the implementation of this algorithm.

3.3.2 Algorithm

Let D be an $N \times M$ matrix, where N is the number of pixels of face image and M the number of face images. D is made of one person’s face images under different lighting conditions. We choose one person’s face images in Yale B Face database with the frontal pose but systemically sampled lighting conditions for building global lighting space. Then we compress these images by Singular Value Decomposition (SVD). Let $V = [v_1, v_2, \dots, v_k]$ be first K eigenvectors of D . Using the texture mapping technique in quotient image, we have the QI for each face image I_i in the database by the ratio of I_i and $V \bullet l_i$,

$$Q_i = \frac{I_i}{V \bullet l_i} \quad (15)$$

$$f(l_i) = \min \|I_t - V \bullet l_i\| \quad (16)$$

where l_i is the estimated lighting by minimizing equation 16.

For any test image I_t with esstimated illumination l_t by Equation 16, we can transfer all face images in the database into the same illumination condition as the input I_t by

$$I_{synti} = Q_i V \bullet l_t \quad (17)$$

Our recognition is carried out among the input image I_t and the I_{syni} , and I_t belongs to class i if it has shortest distance to I_{syni} . Though our algorithm has a similar form as that of quotient image algorithm, our subspace is more complex and lighting l_t is not real light. Moreover the estimated lighting l_t contains shading and shadow information instead of only shading information in quotient image method.

3.4 Self-Quotient Image (S-QI)

In S-QI, a smoothed version of the input image is computed and S-QI is defined as the ratio between the input and its smooth version. Situations with shadows are also considered in our analysis.

3.4.1 Analysis

We define the self-quotient image as an intrinsic property of face images of a person.

Definition 1: (Self-Quotient Image) Self-Quotient image Q of image I is defined by

$$Q = \frac{I}{\hat{I}} = \frac{I}{F * I} \quad (18)$$

where $*$ is convolution operation, \hat{I} is the smoothed I and F is the smoothing kernel.

We call Q Self-Quotient Image because it derives from one image and has the same quotient form as that of quotient image method. We will demonstrate in the following part of this subsection that self-quotient image has similar illumination invariant form as that of quotient image.

The self-quotient image has illumination invariant properties, as demonstrated below using the Lambertian model but with shadow. When shadows present, the Lambertian model of Equation 1 with shadows can be represented as

$$I = \max(\rho n^T \bullet s, 0) \quad (19)$$

In the following, we demonstrate using this Lambertian model that the S-QI is almost illumination invariant, which is an important property for us to develop effective robust face recognition.

Roughly speaking, a scene may consist of the following three types of regions:

- Region 1: no shadow, small shape variation, i.e. surface normal can be regarded constant within the region.
- Region 2: no shadow, big shape variation, i.e. surface normal varies significantly within the region.
- Region 3: shadow regions.



Figure 3: De-shadow effects of S-QI

Now let's discuss these three regions separately.

Region 1:

In this case, $n^T(u, v) \approx C_1$, where C_1 is a constant. Then we have:

$$Q(u, v) = \frac{I(u, v)}{\hat{I}(u, v)} \approx \frac{\rho C_1}{\rho(u, v) * F C_1} = \frac{\rho(u, v)}{\rho(u, v) * F} \quad (20)$$

In this case, Q is approximately illumination free and depends only on the albedo of the face.

Region 2:

In this case, $n^T(u, v)$ is not a constant. The S-QI is given by:

$$Q(u, v) = \frac{I(u, v)}{\hat{I}(u, v)} = \frac{\rho(u, v) n^T(u, v) \bullet s}{F * [\rho(u, v) n^T(u, v) \bullet s]} \quad (21)$$

In such regions, Q depends on the surface normal, albedo and illumination.

Region 3:

In these regions, the grey value of image is relatively low and does not vary to a great deal. Let us assume that light is uniformly distributed from all directions in shadow regions, i.e. for any normal n all the visible lights form a semi-hemisphere. Therefore, the summation of the dot products between n and s_i is constant in such regions:

$$n^T(u, v) \bullet \sum_{i=1}^{\infty} s_i(u, v) = \sum_{i=1}^{\infty} n^T(u, v) \bullet s_i(u, v) = C_2 \quad (22)$$

where C_2 is a constant and $\sum_{i=1}^{\infty} \|s_i\| \ll \|s\|$. Thus, $I(u, v)$ in shadow regions can be written as $I(u, v) \approx C_2 \rho(u, v)$. Then we have:

$$Q(u, v) \approx \frac{I(u, v)}{\hat{I}(u, v)} = \frac{\rho(u, v) C_2}{\rho(u, v) * F C_2} = \frac{\rho(u, v)}{\rho(u, v) * F} \quad (23)$$

Similar to Region 1, the S-QI in this region is also illumination-free, in other words, the S-QI removes the shadow effect, as shown in Figure 3. Although the analysis is based on the Lambertian model of point illumination, it is also valid for other types of illumination sources. This is because any illumination can be expressed as a linear combination of L point illumination sources:

$$I = \rho n^T \bullet s = \rho n^T \bullet \sum_{i=1}^L s_i \quad (24)$$

If we replace the point lighting source s in Regions 1 - 3 with a series of point lights, the analytic results still hold.

The above analysis shows the following two properties of the self-quotient image: (1) The algorithm is robust to illumination variation for case 1 and 3. (2) Q is not the expected reflectance as in Retinex, but the albedo ratio in case 1 and case 3 and illumination dependent image ratio in case 2.

For face recognition, if we can ensure that the filter’s kernel size is small enough compared with face surface normal n^T ’s variation, the self-quotient image will be illumination free as previously analyzed. However, when the filter’s kernel size is too small, Q will approach one and the albedo information is lost.

The advantages of the self-quotient method as opposed to the original quotient image is summarized as follows: (1) The alignment between image I and its smoothed version \hat{I} is automatically perfect, and hence it does not need an alignment procedure. (2) No training images are needed for the estimation of the lighting direction because the lighting fields of I and \hat{I} are similar. (3) the self-quotient image is good at removing shadows; whereas in the previous approaches [1]-[4], the shadow problem was either ignored or was solved by complex 3D rendering. (4) Lighting sources can be any type.

Note that the property of Q is dependant on the kernel size. If the kernel size of F is too small, Q will approximate to one and albedo information will be severely reduced. If the kernel size of F is too large, there will appear halo effects near step-edge region. We use the multi-scale technique to make the result more robust, and in practice, we choose kernel sizes to take more care smoother regions.

3.4.2 Algorithm

Though Jobson’s [6][7] filter is very simple, it is an isotropic one, which creates halo effects around edge region. Gross and Brajovie’s [8] anisotropic filter, which can reduce the halo effect, is an iterative one. For real time application, this method is too computation expensive. The filter F used in our algorithm is weighed Gaussian.

$$F = WG \quad (25)$$

where W is the weight and G is Gaussian kernel. Let ω be the convolution region. We divide the convolution region into two sub-regions M_1 and M_2 with respect to a threshold τ . Assuming that there are more pixels in M_1 than in M_2 and τ is calculated by $\tau = \text{mean}(I_\Omega)$, for the two sub-regions, W has corresponding value.

$$W(u, v) = \begin{cases} 0 & I(u, v) \in M_2 \\ 1 & I(u, v) \in M_1 \end{cases}$$

If the convolution image region is smooth, ie little gray value variation (non-edge region), there is also little difference between the smoothing the whole region and part



Figure 4: Weighed Gaussian Filter



Figure 5: CMU Face Database

of the region. If there is large gray value variation in convolution region, ie. edge region, the threshold can divide the convolution region into two parts M_1 and M_2 along the edge and the filter kernel will convolute only with the large part M_1 , which contains more pixels. Therefore the halo effects can be significantly reduced by the weighed Gaussian kernel. The formation of this filter is shown in Figure 4.

4 Experiments and Discussion

Experiments are performed to evaluate NPL-QI and S-QI for face recognition, using Yale face B database [12] and CMU PIE face database [21]. Frontal face images with lighting variations are selected from the two face databases to reduce the image changes only due to lighting variations. There are 68 subjects in CMU PIE and we select the frontal face images which are taken under 20 different illuminations without background lighting for each subject, shown in Figure 5. There are 640 images (10 subjects with 64 images each) from Yale B.

The eyes, nose and mouth are located manually for each image, and the face is then aligned and cropped. The PCA and original QI methods are also included as the baselines, in which the PCA (60 dimensional) is learned by using all the examples from either PIE or Yale B data sets.

Figure 7 show some results of S-QI based illumination normalization. We can see that the convolution based anisotropic filtering is very effective in smoothing the noisy image without blurring the step edge and shadows are removed.

For the PIE data set, the leave-one-out scheme is used,

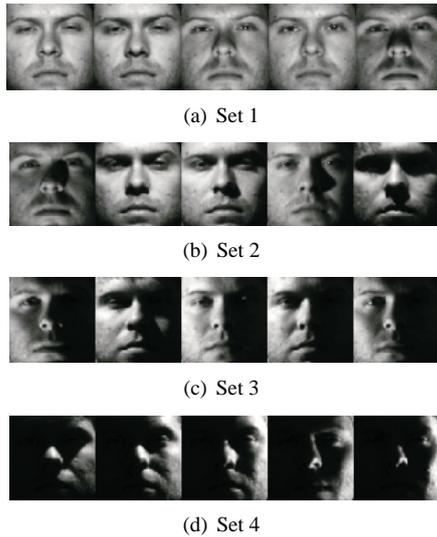


Figure 6: Four Yale B Sets

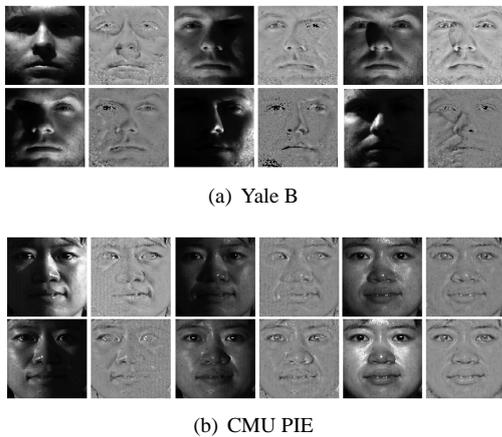


Figure 7: Example results of S-QI illumination removal

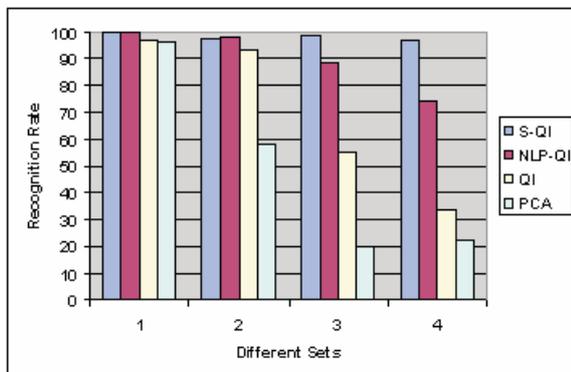


Figure 8: Recognition Results on Yale B Face Database

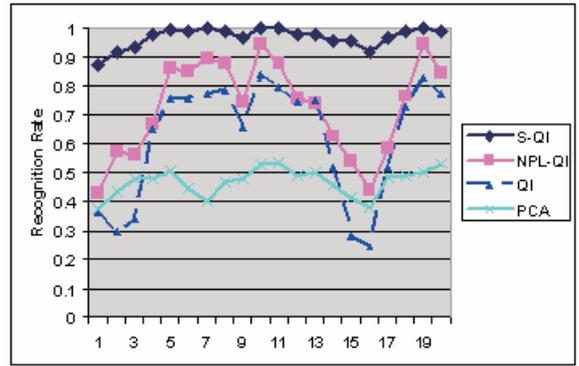


Figure 9: Recognition Results on CMU PIE Face Database

ie each image as template in turn and the others as test images. The results are compared in Figure 9 for the 20 different leave-one-out partitions. For the Yale B data set, the images are divided into 4 subsets of increasing illumination angles, and only the frontal illuminated images are used as the templates. Figure 6 display part of one person's images in the 4 sets. The results are shown in Figure 8 for the 4 different data sets.

Compared with PCA and original QI, both of our new algorithms, S-QI and NLP-QI can significantly improve the recognition rate both in CMU PIE and Yale B face database. It seems that S-QI is more effective than NLP-QI in illumination normalization for face recognizer under our experimental scenario. Also from the recognition results, we can see that the extreme illumination conditions in Yale B set 2 and 3, and CMU image 1 and 15, do have obvious effects on both of our two new methods.

5 Conclusion

A generalized QI framework based on previous works is presented. This unified framework explains the essence of previous QI-based [1, 2], Retinex-based [4],[6]-[8]] and image ratio-based [3, 5] algorithms without any assumption of illumination type and absence of shadow. Under this framework, we derive two new algorithms, NLP-QI and S-QI. These algorithms extend the original QI from point lighting source to any type of lightings, without restrictions on shadows. Compared with the baseline algorithms of original QI and PCA, the two algorithms demonstrate significant performance improvement.

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